1- Use the equation for the height of capillary rise (h):

$$h = \frac{2\sigma\cos(\alpha)}{\rho_w g r}$$

where σ is surface tension [MT^{-2}], ρ_w is the density of water [ML^{-3}], α is contact angle between solid and the air-water interface, g is the acceleration due to gravity [LT^{-1}], and r is the radius of the capillary tube [L].

a. Given α = 0, ρ_w=998 kg m^{-3}, σ = 7.27×10^{-2} N m^{-1} (or kg s^{-2}), and g = 9.81 m s^{-2} use the MS excel and plot the height of rise of water for cylinders with radius equal to 0.001, 0.005, 0.01, 0.05, 0.1, 0.5, and 1 mm.

b. Analyze and briefly explain the effect of the contact angle, α and the temperature of water, T on the height of capillary rise, h by using the following values:

α (contact angle)	0	45	90
T (°C)	10	20	30

[Hint_1: You should deliver with two graphs, each including three lines. In the first plot use α = 0 and compare the effect of temperature of water, and in the second plot use T = 20 °C and compare the effect of contact angle. Use g = 9.81 m s^{-2}]

[Hint_2: Section 2.4 of chapter 1 of the text book can assist for better understanding of the concept]

2- Use van Genuchten-Mualem model:

$$S_e = [1 + (\alpha h)^n]^{-m} \tag{1}$$

where S_e denotes the effective (relative) saturation (It is also called degree of saturation) [-], $S_e = (\theta - \theta_r)/(\theta_s - \theta_r)$, with θ_s and θ_r are the saturated and residual wetting fluid saturation [L^3 L^{-3}], respectively; h is soil matric potential, α [L^{-1}], n[-], l are fitting parameters, that are determined by the air entry value, the pore-size distribution, and the soil connectivity (also referred to tortuosity), respectively. It is also assumed that m = 1-1/n.

$$K_r = \frac{K(S_e)}{K_s} = S_e^{\,l}[1 - (1 - S_e^{\frac{1}{m}})^m]^2 \tag{2}$$

where K_s denotes saturated hydraulic conductivity [L T^{-1}], K_r represent the relative hydraulic conductivity [-], $K(S_e)$ is the hydraulic conductivity at degree of saturation [L T^{-1}], and $l = 0.5$ [-]. Given $\theta_r = 0.1$ cm^3 cm^{-3}, $\theta_s = 0.5$ cm^3 cm^{-3}, $n = 2.0$, $K_s = 1$ cm h^{-1}, and $\alpha = 0.01$ cm^{-1}:
(Choose the range of h from zero to 15000 cm)

a. Write computer program (MS Excel or any other programming language) to generate data points for $\theta(h)$ and $K(\theta)$;
b. Derive functional expression for $C(h)$;
   (Note that $C = d\theta/dh$)
   Use program of (a) to also generate data for $C(h)$
c. Plot curves for $S_e(h)$, $C(S_e)$, $K(h)$and $K(S_e)$;
d. Analyze and briefly explain the effect of parameters α, n, and l on $S_e(h)$ and $K_r(h)$ by using the following values:

Problem set#1
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α	0.01	0.05	0.1
n	1.5	3	5
l	0.5	-1	1

[Hint_1: For both $S_e(h)$ and $K_r(h)$ you should generate three lines in each of three graphs. For example, keep α and n constant and generate data points of $S_e(h)$ and $K_r(h)$ for different l, then plot these data points in one graph.]

[Hint_2: Section 2.5 and 2.9.4 of chapter 1 of the text book can assist for better understanding of the concept]