

## Modeling of Water Flow and Solute Transport in the Vadose Zone

# Introduction to Hydrus Models

---

**Jirka Šimůnek<sup>1</sup>,**  
 Rien van Genuchten<sup>2</sup>, Miroslav Šejna<sup>2</sup>, Diederik Jacques<sup>4</sup>, and Navin Twarakavi<sup>5</sup>

<sup>1</sup>Department of Environmental Sciences, UC Riverside, CA  
<sup>2</sup>Department of Mechanical Engineering, Federal University of Rio de Janeiro, Brazil  
<sup>3</sup>PC-Progress, Ltd., Prague, Czech Republic  
<sup>4</sup>Belgian Nuclear Research Centre (SCK•CEN), Mol, Belgium  
<sup>5</sup>Department of Agronomy and Soils, Auburn University, Auburn, AL

1

---

---

---

---

---

---

---


---

---

---

## Program Developers – Jirka Šimůnek

A Professor of Hydrology with the Department of Environmental Sciences of the University of California, Riverside. Received an M.S. in Civil Engineering from the Czech Technical University, Prague, Czech Republic, and a Ph.D. in Water Management from the Czech Academy of Sciences, Prague.



Expertise in numerical modeling of subsurface water flow and solute transport processes, equilibrium and nonequilibrium chemical transport, multicomponent major ion chemistry, field-scale spatial variability, and inverse procedures for estimating soil hydraulic and solute transport parameters.

He has authored and coauthored over 190 peer-reviewed journal publications, over 20 book chapters, and 2 books. His numerical HYDRUS models are used by virtually all scientists, students, and practitioners modeling water flow, chemical movement, and heat transport through variably saturated soils. Dr. Šimůnek is a recipient of the Soil Science Society of America's Don and Betty Kirkham Soil Physics Award, Fellow and the past chair of the Soil Physics (S1) of Soil Sciences Society of America. He is an associate editor of Vadose Zone Journal, Journal of Hydrological Sciences, Journal of Hydrology and Hydromechanics, and a past AE of Water Resources Research.

2

---

---

---

---

---

---

---


---

---

---

## Program Developers – Rien van Genuchten

A soil physicist originally with the George E. Brown, Jr. Salinity Laboratory, USDA, ARS, Riverside, CA. Received a B.S. and M.S. in irrigation and drainage from Wageningen University in The Netherlands, and a Ph.D. in soil physics from New Mexico State University.



Dr. van Genuchten is a recipient of the Soil Science Society of America's Don and Betty Kirkham Soil Physics Award, of the EGU Dalton Medal, and fellow of the Soil Science Society of America, American Society of Agronomy, American Geophysical Union and American Association for the Advancement of Sciences. Founding Editor of the Vadose Zone Journal. Currently with the University of Rio de Janeiro, Brazil.

Research on variably-saturated water flow and contaminant transport, analytical and numerical modeling, nonequilibrium transport, preferential flow, characterization and measurement of the unsaturated soil hydraulic functions, salinity management, and root-water uptake. Most often referenced researcher in the field of Soil Physics. Dr. van Genuchten is probably best known for the theoretical equations he developed for the nonlinear constitutive relationships between capillary pressure, water content and the hydraulic conductivity of unsaturated media.

3

---

---

---

---

---

---

---

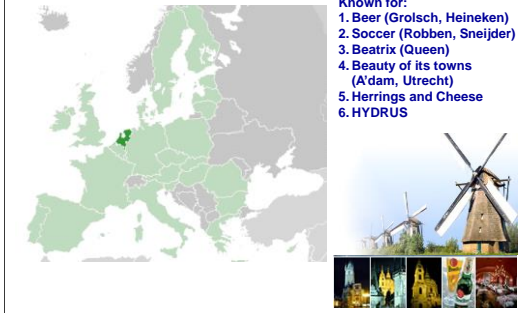
---

---

---



## Holland (Netherlands) + Belgium



7

---

---

---

---

---

---

---

---

---

---

## Subsurface Flow and Transport

### Hydrus-1D and Hydrus (2D/3D)

- numerical models that simulate

a) Water flow:

b) Solute Transport

c) Heat Transport

in unsaturated, partially saturated, or fully saturated one-, two-, or three-dimensional porous media, i.e., in nonuniform soils

8

---

---

---

---

---

---

---

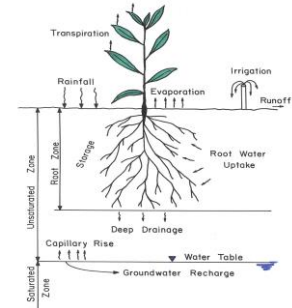
---

---

---

## Agricultural Applications

- ◆ Precipitation
- ◆ Irrigation
- ◆ Runoff
- ◆ Evaporation
- ◆ Transpiration
- ◆ Root Water Uptake
- ◆ Capillary Rise
- ◆ Deep Drainage
- ◆ Fertigation
- ◆ Pesticides
- ◆ Fumigants
- ◆ Colloids
- ◆ Pathogens



9

---

---

---

---

---

---

---

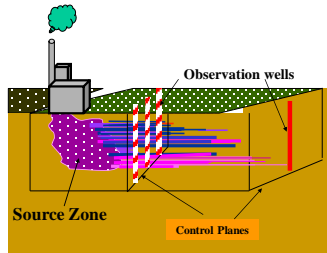
---

---

---

## Industrial Applications

- ◆ Industrial Pollution
- ◆ Municipal Pollution
- ◆ Landfill Covers
- ◆ Waste Repositories
- ◆ Radioactive Waste Disposal Sites
- ◆ Remediation
- ◆ Brine Releases
- ◆ Contaminant Plumes
- ◆ Seepage of Wastewater from Land Treatment Systems



10

---

---

---

---

---

---

---

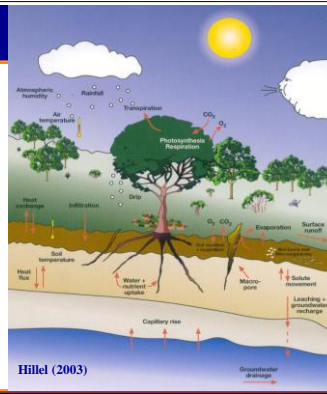
---

---

---

## Environmental Applications

- ◆ Ecological Apps
- ◆ Carbon Storage and Fluxes
- ◆ Heat Exchange and Fluxes
- ◆ Nutrient Transport
- ◆ Soil Respiration
- ◆ Microbiological Processes
- ◆ Effects of Climate Change
- ◆ Riparian Systems
- ◆ Stream-Aquifer Interactions



11

---

---

---

---

---

---

---

---

---

---

## Governing Equations

Variably-Saturated Water Flow (**Richards Equation**)

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[ K(h) \frac{\partial h}{\partial z} - K(h) \right] - S$$

Heat Movement

$$\frac{\partial C_p(\theta)T}{\partial t} = \frac{\partial}{\partial z} \left[ \lambda(\theta) \frac{\partial T}{\partial z} \right] - C_w \frac{\partial qT}{\partial z} - C_w ST$$

Solute Transport (**Convection-Dispersion Equation**)

$$\frac{\partial(\rho s)}{\partial t} + \frac{\partial(\theta c)}{\partial t} = \frac{\partial}{\partial z} \left( \theta D \frac{\partial c}{\partial z} - qc \right) - \phi$$

12

---

---

---

---

---

---

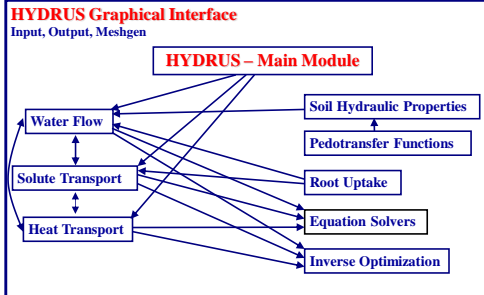
---

---

---

---

## HYDRUS –Modular Structure



13

---

---

---

---

---

---

---

---

---

---

## HYDRUS Software Packages

- Water Flow:**
- ◆ Richards equation for variably-saturated water flow
  - ◆ Various models of soil hydraulic properties
  - ◆ Hysteresis
  - ◆ Sink term to account for water uptake by plant roots
- Heat Transport:**
- ◆ Conduction and convection with flowing water
- Solute Transport:**
- ◆ Convective-dispersive transport in the liquid phase, diffusion in the gaseous phase
  - ◆ Nonlinear nonequilibrium reactions between the solid and liquid phases
  - ◆ Linear equilibrium reactions between the liquid and gaseous phases
  - ◆ Zero-order production
  - ◆ First-order degradation reactions
  - ◆ Physical nonequilibrium solute transport

14

---

---

---

---

---

---

---

---

---

---

## The HYDRUS Software Packages

- ◆ **Variably-Saturated Flow (Richards Eq.)**
- ◆ Root Water Uptake (water and salinity stress)
- ◆ Solutes Transport (decay chains, ADE)
  - Sorption (linear and nonlinear)
  - Chemical Nonequilibrium
  - Physical Nonequilibrium
- ◆ Heat Transport
- ◆ Parameter Estimation
- ◆ Interactive Graphics-Based Interface
- ◆ Additional Modules

18

---

---

---

---

---

---

---

---

---

---

## Water Flow in Soils

◆ **Groundwater flow (Darcy's Law)**

$$q = K_s \frac{\Delta(P+z)}{\Delta L} = -K_s \frac{dH}{dz}$$

◆ **Unsaturated water flow (Darcy-Buckingham Law)**

$$q = K(h) \frac{\Delta(h+z)}{\Delta L} = -K(h) \frac{dH}{dz}$$

$H$  - sum of the matric ( $h$ ) and gravitational ( $z$ ) head  
( $H=h+z$ )

19

---

---

---

---

---

---

---

---

---

---

## Water Flow - Richards Equation

The governing flow equation for one-dimensional isothermal Darcian flow in a variably-saturated isotropic rigid porous medium:

$$\frac{\partial \theta(h)}{\partial t} = -\frac{\partial q}{\partial z} - S(h)$$

$$\frac{\partial \theta(h)}{\partial t} = \frac{\partial}{\partial z} \left[ K(h) \frac{\partial h}{\partial z} + K(h) \right] - S(h)$$

- $\theta$  - volumetric water content [ $L^3L^{-3}$ ]
- $h$  - pressure head [L]
- $K$  - unsaturated hydraulic conductivity [ $LT^{-1}$ ]
- $z$  - vertical coordinate positive upward [L]
- $t$  - time [T]
- $S$  - root water uptake [ $T^{-1}$ ]

20

---

---

---

---

---

---

---

---

---

---

## Water Flow - Richards Equation

The governing flow equation for two-dimensional isothermal Darcian flow in a variably-saturated isotropic rigid porous medium:

$$\frac{\partial \theta(h)}{\partial t} = \frac{\partial}{\partial x_i} \left[ K(h) \left( K_{ij}^A \frac{\partial h}{\partial x_j} + K_{iz}^A \right) \right] - S(h)$$

- $\theta$  - volumetric water content [ $L^3L^{-3}$ ]
- $h$  - pressure head [L]
- $K$  - unsaturated hydraulic conductivity [ $LT^{-1}$ ]
- $K_{ij}^A$  - components of an anisotropy tensor [-]
- $x_i$  - spatial coordinates [L]
- $z$  - vertical coordinate positive upward [L]
- $t$  - time [T]
- $S$  - root water uptake [ $T^{-1}$ ]

21

---

---

---

---

---

---

---

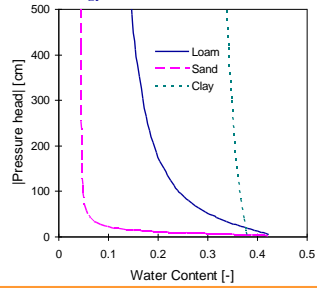
---

---

---

## Retention Curve, $\theta(h)$

Soil-water characteristic curve  
Characterizes the energy status of the soil water



22

---

---

---

---

---

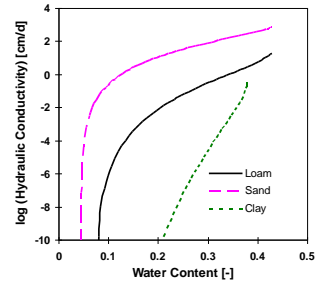
---

---

---

## Hydraulic Conductivity, $K(\theta)$

- characterizes resistance of porous media to water flow



23

---

---

---

---

---

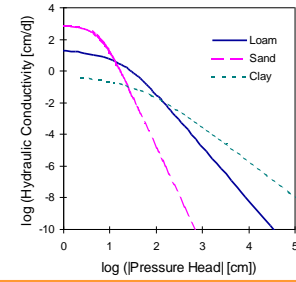
---

---

---

## Hydraulic Conductivity, $K(h)$

- characterizes resistance of porous media to water flow



24

---

---

---

---

---

---

---

---





## Richards Equation - Assumptions

- ▶ Effect of **air phase** is neglected
- ▶ Darcy's equation is valid at very low and very high **velocities**
- ▶ **Osmotic gradients** in the soil water potential are negligible
- ▶ **Fluid density** is independent of solute concentration
- ▶ Matrix and fluid **compressibilities** are relatively small

28

---

---

---

---

---

---

---

---

---

---

## Richards Equation - Complications

- ▶ **Hysteresis** in the soil water retention function
- ▶ Extreme **nonlinearity** of the hydraulic functions
- ▶ Lack of accurate and cheap methods for **measuring** the hydraulic properties
- ▶ Extreme **heterogeneity** of the subsurface
- ▶ Inconsistencies between **scale** at which the hydraulic and solute transport parameters are measured, and the scale at which the models are being applied

29

---

---

---

---

---

---

---

---

---

---

## Boundary Conditions (System-Independent)

- **Pressure head (Dirichlet type) boundary conditions:**

$$h(z,t) = h_0(z,t) \quad \text{for } z=0 \quad \text{or } z=L$$

- **Flux (Neumann type) boundary conditions:**

$$-K \left( \frac{\partial h}{\partial z} + 1 \right) = q_0(z,t) \quad \text{for } z=0 \quad \text{or } z=L$$

- **Gradient boundary conditions:**

$$\frac{\partial h}{\partial z} = 1 \quad \text{for } z=L$$

30

---

---

---

---

---

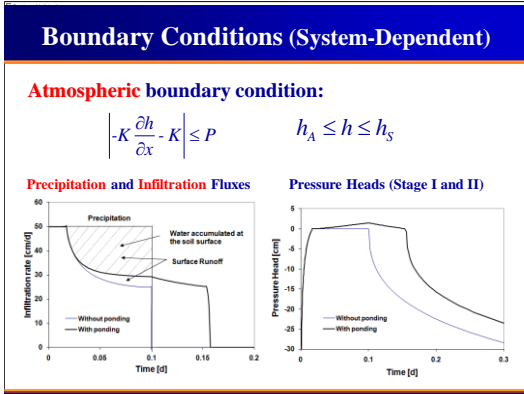
---

---

---

---

---



31

---

---

---

---

---

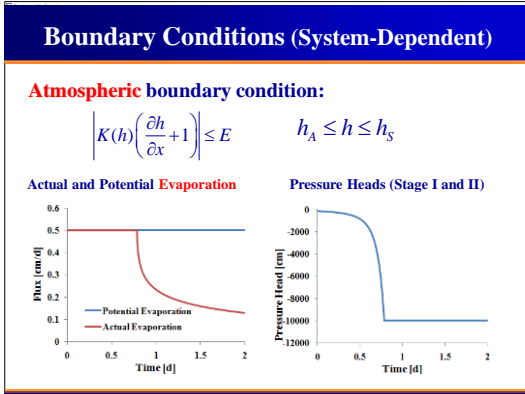
---

---

---

---

---



32

---

---

---

---

---

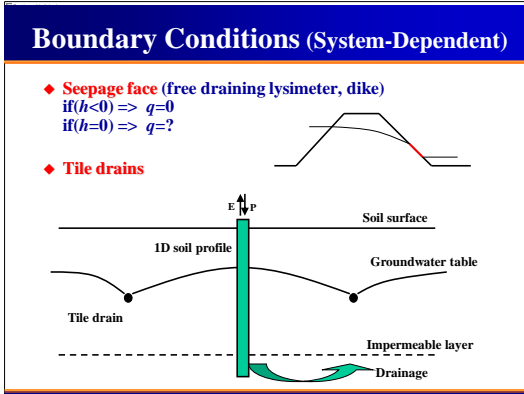
---

---

---

---

---



33

---

---

---

---

---

---

---

---

---

---

## The HYDRUS Software Packages

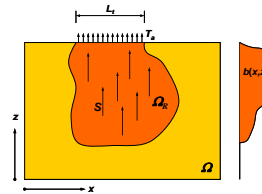
- ◆ Variably-Saturated Flow (Richards Eq.)
- ◆ **Root Water Uptake (water and salinity stress)**
- ◆ Solutes Transport (decay chains, ADE)
  - Sorption (linear and nonlinear)
  - Chemical Nonequilibrium
  - Physical Nonequilibrium
- ◆ Heat Transport
- ◆ Parameter Estimation
- ◆ Interactive Graphics-Based Interface
- ◆ Additional Packages

34

## Root Water Uptake

Feddes et al. [1978]

$$S_p(z, t) = b(z)T_p$$

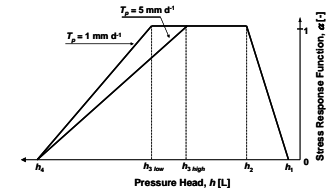


- $b$  normalized water uptake distribution [L<sup>-1</sup>]
- $S_p$  potential root water uptake [T<sup>-1</sup>]
- $T_p$  potential transpiration [LT<sup>-1</sup>]

35

## Root Water Uptake

Feddes et al. [1978]  $S(z, t) = \alpha(h)S_p(z, t) = \alpha(h)b(z)T_p$



- $b$  normalized water uptake distribution [L<sup>-1</sup>]
- $S_p$  potential root water uptake [T<sup>-1</sup>]
- $S$  actual root water uptake [T<sup>-1</sup>]
- $T_p$  potential transpiration [LT<sup>-1</sup>]
- $\alpha$  stress response function [-]

36

---

---

---

---

---

---

---

---

---

---



---

---

---

---

---

---

---

---

---

---



---

---

---

---

---

---

---

---

---

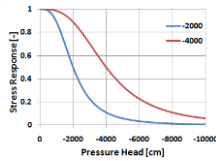
---

## Stress Response Functions

Water and solute stress:

$$\alpha(h, h_o) = \frac{1}{1 + \left(\frac{h + h_o}{h_{50}}\right)^p}$$

$$\alpha(h, h_o) = \frac{1}{1 + \left(\frac{h}{h_{50}}\right)^{p_1}} * \frac{1}{1 + \left(\frac{h_o}{h_{p50}}\right)^{p_2}}$$



- $\alpha$  stress response function [-]
- $h$  pressure head [L]
- $h_o$  osmotic head [L]
- $h_{50}$  pressure head at which water extraction rate is reduced by 50% [L]
- $h_{p50}$  ditto for osmotic head [L]
- $p_1, p_2$  experimental constants [-] (=3)

37

---

---

---

---

---

---

---

---

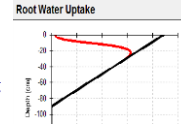
---

---

## Transpiration Rates

$$T_p = \int_{L_R} S_p(h, z) dz$$

$$T_a = \int_{L_R} S(h, z) dz = T_p \int_{L_R} a(h, z) b(z) dz$$



- $b$  normalized water uptake distribution [L<sup>-1</sup>]
- $\alpha$  stress response function [-]
- $S_p$  potential root water uptake [T<sup>-1</sup>]
- $S$  actual root water uptake [T<sup>-1</sup>]
- $T_p$  potential transpiration [LT<sup>-1</sup>]
- $T_a$  actual transpiration [LT<sup>-1</sup>]

38

---

---

---

---

---

---

---

---

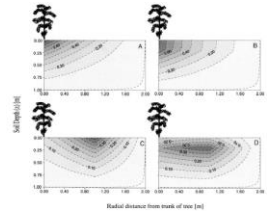
---

---

## Spatial Root Distribution Function

$$b(x, y, z) = \left(1 - \frac{x}{X_m}\right) \left(1 - \frac{y}{Y_m}\right) \left(1 - \frac{z}{Z_m}\right) e^{-\left[\left(\frac{p_1}{X_m}\right)^2 |x|^2 - 1\right] - \left[\left(\frac{p_2}{Y_m}\right)^2 |y|^2 - 1\right] - \left[\left(\frac{p_3}{Z_m}\right)^2 |z|^2 - 1\right]}$$

$$b(x, z) = \left(1 - \frac{z}{Z_m}\right) \left(1 - \frac{x}{X_m}\right) e^{-\left[\left(\frac{p_1}{X_m}\right)^2 |x|^2 - 1\right] - \left[\left(\frac{p_3}{Z_m}\right)^2 |z|^2 - 1\right]}$$



(Vrugt et al., 2001)

39

---

---

---

---

---

---

---

---

---

---

## Root Growth

$$L_R(t) = L_m f(t)$$

$$f(t) = \frac{L_0}{L_0 + (L_m - L_0)e^{-rt}}$$

- $L_R$  rooting depth [L]
- $L_0$  initial rooting depth [L]
- $L_m$  maximum rooting depth [L]
- $f$  root growth coefficient (Verhulst-Pearl logistic function)
- $r$  growth rate [ $T^{-1}$ ]

40

---

---

---

---

---

---

---

---

---

---

## The HYDRUS Software Packages

- ◆ Variably-Saturated Flow (Richards Eq.)
- ◆ Root Water Uptake (water and salinity stress)
- ◆ **Solutes Transport (decay chains, ADE)**
  - Sorption (linear and nonlinear)
  - Chemical Nonequilibrium
  - Physical Nonequilibrium
- ◆ Heat Transport
- ◆ Parameter Estimation
- ◆ Interactive Graphics-Based Interface
- ◆ Additional Modules

41

---

---

---

---

---

---

---

---

---

---

## Solute Transport - Convection-Dispersion Equation

One-dimensional chemical transport during transient water flow in a variably saturated rigid porous medium

$$\frac{\partial(\theta c)}{\partial t} + \frac{\partial(\rho s)}{\partial t} = \frac{\partial}{\partial z} \left( \theta D \frac{\partial c}{\partial z} - qc \right) - \phi$$

- $c$  - solution concentration [ $ML^{-3}$ ]
- $s$  - adsorbed concentration [ $MM^{-1}$ ]
- $\theta$  - water content [ $L^3L^{-3}$ ]
- $\rho$  - soil bulk density [ $ML^{-3}$ ]
- $D$  - dispersion coefficient [ $L^2T^{-1}$ ]
- $q$  - volumetric flux [ $LT^{-1}$ ]
- $\phi$  - rate constant representing reactions [ $ML^{-3}T^{-1}$ ]

42

---

---

---

---

---

---

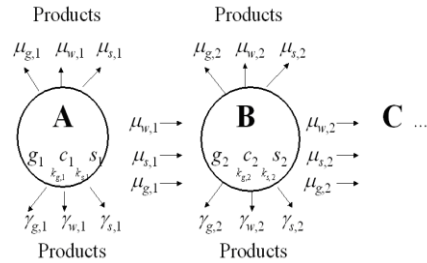
---

---

---

---

## General Structure of the System of Solutes



43

---

---

---

---

---

---

---

---

---

---

## HYDRUS – Solute Transport

- ◆ **Transport of single ions**
- ◆ **Transport of multiple ions (sequential first-order decay)**
  - ▶ **Radionuclides:**  $^{238}\text{Pu} \rightarrow ^{234}\text{U} \rightarrow ^{230}\text{Th} \rightarrow ^{226}\text{Ra}$
  - ▶ **Nitrogen:**  $(\text{NH}_2)_2\text{CO} \rightarrow \text{NH}_4^+ \rightarrow \text{NO}_2^- \rightarrow \text{NO}_3^-$
  - ▶ **Pesticides:** aldicarb (oxime)  $\rightarrow$  sulfone (sulfone oxime)  $\rightarrow$  sulfoxide (sulfoxide oxime)
  - ▶ **Chlorinated Hydrocarbons:** PCE  $\rightarrow$  TCE  $\rightarrow$  c-DCE  $\rightarrow$  VC  $\rightarrow$  ethylene
  - ▶ **Pharmaceuticals, hormones:** Estrogen (17bEstradiol  $\rightarrow$  Estrone  $\rightarrow$  Estriol), Testosterone
  - ▶ **Explosives:** TNT ( $\rightarrow$  4HADNT  $\rightarrow$  4ADNT  $\rightarrow$  TAT), RDX HMX

44

---

---

---

---

---

---

---

---

---

---

## Governing Solute Transport Equations

$$\frac{\partial(\theta c)}{\partial t} + \frac{\partial(\rho s)}{\partial t} = \frac{\partial}{\partial z} \left( \theta D \frac{\partial c}{\partial z} - qc \right) - \phi$$

$$\frac{\partial \theta c_k}{\partial t} + \frac{\partial \rho s_k}{\partial t} + \frac{\partial a g_k}{\partial t} = \frac{\partial}{\partial z} \left( \theta D_k^* \frac{\partial c_k}{\partial z} \right) + \frac{\partial}{\partial z} \left( a D_k^* \frac{\partial g_k}{\partial z} \right) - \frac{\partial q c_k}{\partial z} - (\mu_{w,k} + \mu'_{w,k}) \theta c_k - (\mu_{s,k} + \mu'_{s,k}) \rho s_k - (\mu_{g,k} + \mu'_{g,k}) a g_k + \mu'_{w,k-1} \theta c_{k-1} + \mu'_{s,k-1} \rho s_{k-1} - \mu'_{g,k-1} a g_{k-1} + \gamma_{w,k} \theta + \gamma_{s,k} \rho + \gamma_{g,k} a - S c_{r,k} \quad k \in (2, n_s)$$

$w, s, g$  subscripts corresponding with the liquid, solid and gaseous phases, respectively  
 $c, s, g$  concentration in liquid, solid, and gaseous phase, respectively

45

---

---

---

---

---

---

---

---

---

---

## Solute Transport - Boundary Conditions

- **First-type** (or Dirichlet type) boundary conditions

$$c(x, z, t) = c_0(x, z, t) \quad \text{for } (x, z) \in \Gamma_D$$

- **Third-type** (Cauchy type) boundary conditions

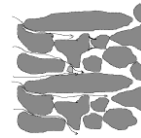
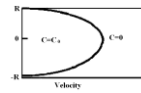
$$-\theta D_{ij} \frac{\partial c}{\partial x_j} n_i + q_i n_i c = q_i n_i c_0 \quad \text{for } (x, z) \in \Gamma_C$$

- **Second-type** (Neumann type) boundary conditions

$$\theta D_{ij} \frac{\partial c}{\partial x_j} n_i = 0 \quad \text{for } (x, z) \in \Gamma_N$$

47

## Solute Transport – Dispersion Coefficient



**Bear [1972]:**

$$\theta D = \lambda / q + \theta D_d \tau$$

- $D_d$  - ionic or molecular diffusion coefficient in free water [ $L^2 T^{-1}$ ]
- $\tau$  - tortuosity factor [-]
- $\lambda$  - longitudinal dispersivity [L]
- $\theta$  - water content [ $L^3 L^{-3}$ ]
- $q$  - Darcy's flux [ $L T^{-1}$ ]

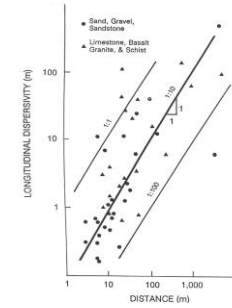
**Millington and Quirk [1961]:**

$$\tau = \frac{\theta^{7/3}}{\theta_s^2}$$

- $\theta_s$  - saturated water content [-]

48

## Dispersivity as a Function of Scale



Gelhar et al. (1985)

50

## The HYDRUS Software Packages

- ◆ Variably-Saturated Flow (Richards Eq.)
- ◆ Root Water Uptake (water and salinity stress)
- ◆ Solutes Transport (decay chains, ADE)
  - Sorption (linear and nonlinear)
  - Chemical Nonequilibrium
  - Physical Nonequilibrium
- ◆ Heat Transport
- ◆ Parameter Estimation
- ◆ Interactive Graphics-Based Interface
- ◆ Additional Modules

51

## Convection-Dispersion Equation

### Linear Adsorption

$$s = K_d c \quad R = 1 + \frac{\rho K_d}{\theta}$$
$$\frac{\partial R\theta c}{\partial t} = \frac{\partial}{\partial x_i} \left( \theta D_{ij} \frac{\partial c}{\partial x_j} - q_i c \right) + \phi$$

- $K_d$  - distribution coefficient [ $L^3M^{-1}$ ]  
 $R$  - retardation factor [-]  
 $s$  - solid phase concentration [ $MM^{-1}$ ]  
 $c$  - liquid phase concentration [ $ML^{-3}$ ]

52

## Nonlinear Equilibrium Adsorption

- HYDRUS assumes nonequilibrium interactions between the solution ( $c$ ) and adsorbed ( $s$ ) concentrations, and equilibrium interaction between the solution ( $c$ ) and gaseous ( $g$ ) concentrations of the solute in the soil system.
- Liquid - Solid: a generalized **nonlinear** (Freundlich-Langmuir) empirical equation

$$s = \frac{k_s c^\beta}{1 + \eta c^\beta}$$

- $k_s, \eta, \beta$  empirical constants

53



## The HYDRUS Software Packages

- ◆ Variably-Saturated Flow (Richards Eq.)
- ◆ Root Water Uptake (water and salinity stress)
- ◆ **Solute Transport (decay chains, ADE)**
  - Sorption (linear and nonlinear)
  - Chemical Nonequilibrium
  - Physical Nonequilibrium
- ◆ Heat Transport
- ◆ Parameter Estimation
- ◆ Interactive Graphics-Based Interface
- ◆ Additional Modules

54

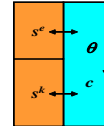
## Nonequilibrium Two-Site Adsorption Model

$$S = S^e + S^k$$

$s^e$  Type-1 sites with instantaneous sorption  
 $s^k$  Type-2 sites with kinetic sorption

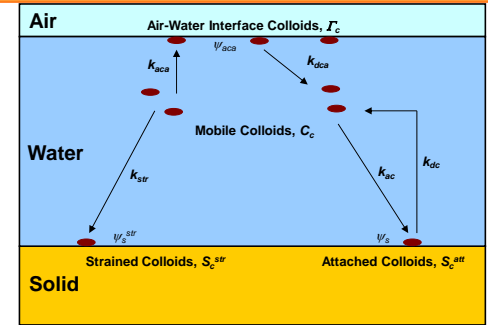
$$\frac{\partial s^k}{\partial t} = \alpha[(1-f)K_d c - s^k]$$

$f$  fraction of exchange sites assumed to be at equilibrium



55

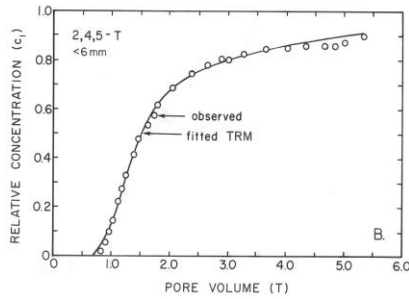
## Colloid, Virus, and Bacteria Transport



56



## Two-Region Physical Nonequilibrium Transport



60

---

---

---

---

---

---

---

---

---

---

## Interaction Among Phases

Liquid - Gas: a **linear** relation (Henry's Law)

$$g = k_g c$$

$k_g$  empirical constant equal to  $(K_H RT_A)^{-1}$   
 $K_H$  Henry's Law constant  
 $R$  universal gas constant  
 $T_A$  absolute temperature

61

---

---

---

---

---

---

---

---

---

---

## Temperature Dependence of Transport and Reaction Coefficients

Most of the diffusion ( $D_w, D_c$ ), distribution ( $k_p, k_g$ ), and reaction rate ( $\gamma_s, \gamma_g, \mu_w, \mu_s, \mu_g, \mu_p, \mu_r$ , and  $\mu_d$ ) coefficients are strongly temperature dependent. HYDRUS assumes that this dependency can be expressed by an Arrhenius equation [Stumm and Morgan, 1981].

$$a_T = a_r \exp \left[ \frac{E(T^A - T_r^A)}{RT^A T_r^A} \right]$$

$a_r, a_T$  coefficient values at a reference absolute temperature,  $T_r^A$ , and absolute temperature,  $T^A$ , respectively  
 $E$  activation energy of the reaction or process

62

---

---

---

---

---

---

---

---

---

---

## The HYDRUS Software Packages

- ◆ Variably-Saturated Flow (Richards Eq.)
- ◆ Root Water Uptake (water stress)
- ◆ Solutes Transport (decay chains, ADE)
  - Sorption (linear and nonlinear)
  - Chemical Nonequilibrium
  - Physical Nonequilibrium
- ◆ Heat Transport
- ◆ **Parameter Estimation**
- ◆ Interactive Graphics-Based Interface
- ◆ Additional Modules

66

## Parameter Estimation with HYDRUS

- Parameter Estimation:**
- Soil hydraulic parameters
  - Solute transport and reaction parameters
  - Heat transport parameters
- Sequence:**
- Independently
  - Simultaneously
  - Sequentially
- Method:**
- Marquardt-Levenberg optimization

67

## Formulation of the Inverse Problem

The problem can be simplified into  
the **Weighted Least-Squares Problem**

$$\Phi(\beta) = \sum_{i=1}^n w_i [q_i^* - q_i(\beta)]^2$$

$w_i$  - weight of a particular measured point

69

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

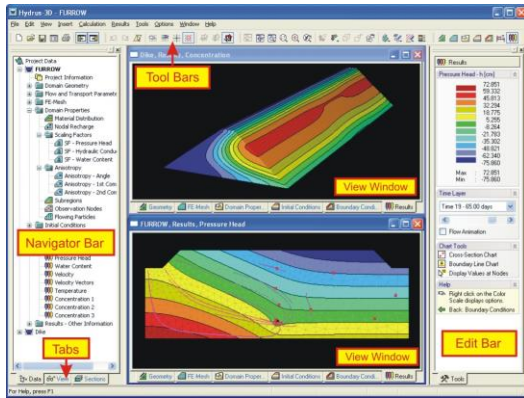
---

---

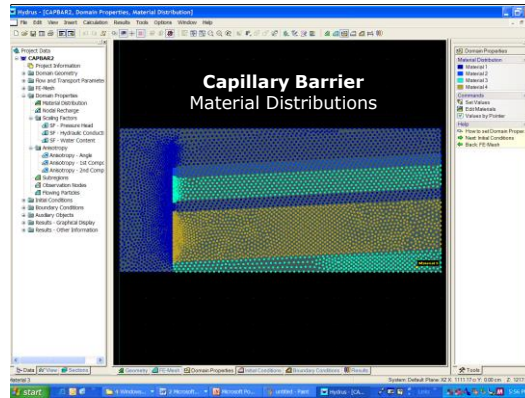
---

---

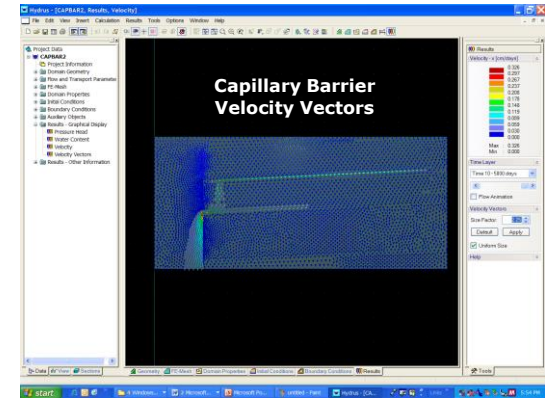




73



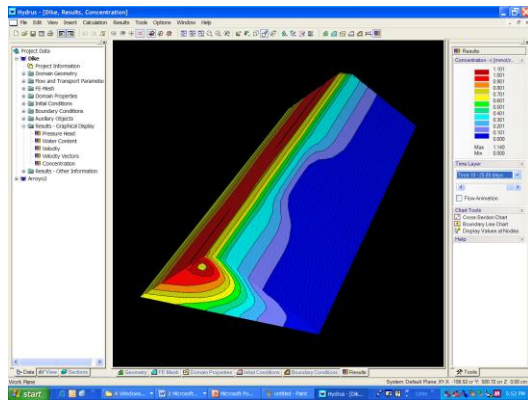
74



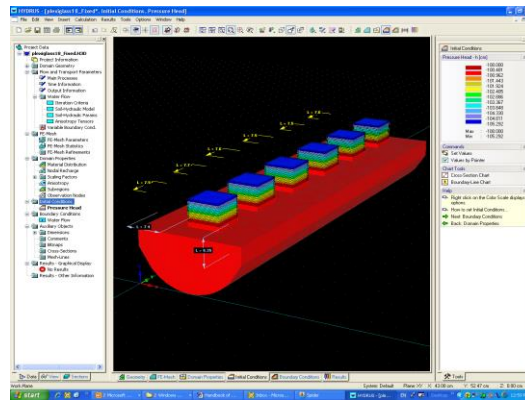
75



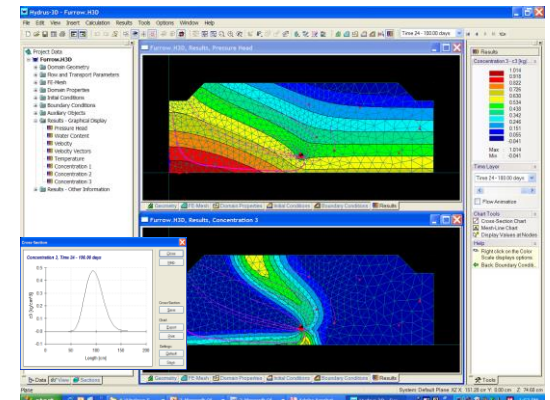




79



80



81





## The HYDRUS Software Packages

- ◆ Variably-Saturated Flow (Richards Eq.)
- ◆ Root Water Uptake (water stress)
- ◆ Solutes Transport (decay chains, ADE)
  - Sorption (linear and nonlinear)
  - Chemical Nonequilibrium
  - Physical Nonequilibrium
- ◆ Heat Transport
- ◆ Parameter Estimation
- ◆ Interactive Graphics-Based Interface
- ◆ Additional Modules

82

## Geochemical Modeling



83

## HYDRUS-1D + UNSATCHEM

- **HYDRUS-1D** (Šimůnek et al., 1998)
  - variably saturated water flow
  - heat transport
  - root water uptake
  - solute transport
- **UNSATCHEM** (Šimůnek et al., 1996)
  - carbon dioxide transport
  - major ion chemistry
    - cation exchange
    - precipitation-dissolution (instantaneous and kinetic)
    - complexation

84

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

## Carbon Dioxide Transport

Šimůnek and Suarez [1993]:

$$\frac{\partial(c_a\theta_a + c_w\theta)}{\partial t} = \frac{\partial}{\partial x_i} \theta_a D_{ij}^g \frac{\partial c_a}{\partial x_j} + \frac{\partial}{\partial x_i} \theta D_{ij}^w \frac{\partial c_w}{\partial x_j} - \frac{\partial}{\partial x_i} q_i c_w - S c_w + P$$

$c_w$  volumetric concentrations of CO<sub>2</sub> in the dissolved phase [L<sup>3</sup>L<sup>-3</sup>]

$c_a$  volumetric concentrations of CO<sub>2</sub> in the gas phase [L<sup>3</sup>L<sup>-3</sup>]

$D_{ij}^g$  effective soil matrix diffusion coefficient of CO<sub>2</sub> in the gas phase [L<sup>2</sup>T<sup>-1</sup>]

$D_{ij}^w$  effective soil matrix dispersion coefficient of CO<sub>2</sub> in the dissolved phase [L<sup>2</sup>T<sup>-1</sup>]

$q_i$  soil water flux [LT<sup>-1</sup>]

$\theta_a$  volumetric air content [L<sup>3</sup>L<sup>-3</sup>]

$P$  CO<sub>2</sub> production rate [L<sup>3</sup>L<sup>-3</sup>T<sup>-1</sup>]

$S c_w$  dissolved CO<sub>2</sub> removed from the soil by root water uptake

85

## HYDRUS-1D + UNSATCHEM

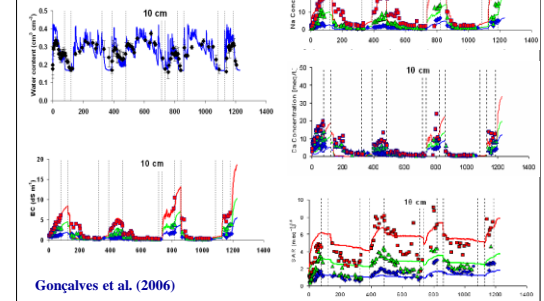
1	Aqueous components	7	Ca <sup>2+</sup> , Mg <sup>2+</sup> , Na <sup>+</sup> , K <sup>+</sup> , SO <sub>4</sub> <sup>2-</sup> , Cl <sup>-</sup> , NO <sub>3</sub> <sup>-</sup>
2	Complexed species	10	CaCO <sub>3</sub> <sup>0</sup> , CaHCO <sub>3</sub> <sup>+</sup> , CaSO <sub>4</sub> <sup>0</sup> , MgCO <sub>3</sub> <sup>0</sup> , MgHCO <sub>3</sub> <sup>+</sup> , MgSO <sub>4</sub> <sup>0</sup> , NaCO <sub>3</sub> <sup>-</sup> , NaHCO <sub>3</sub> <sup>0</sup> , NaSO <sub>4</sub> <sup>-</sup> , KSO <sub>4</sub> <sup>-</sup>
3	Precipitated species	6	CaCO <sub>3</sub> , CaSO <sub>4</sub> · 2H <sub>2</sub> O, MgCO <sub>3</sub> · 3H <sub>2</sub> O, Mg <sub>5</sub> (CO <sub>3</sub> ) <sub>4</sub> (OH) <sub>2</sub> · 4H <sub>2</sub> O, Mg <sub>2</sub> Si <sub>3</sub> O <sub>7,5</sub> (OH) · 3H <sub>2</sub> O, CaMg(CO <sub>3</sub> ) <sub>2</sub>
4	Sorbed species (exchangeable)	4	Ca, Mg, Na, K
5	CO <sub>2</sub> -H <sub>2</sub> O species	7	$P_{CO_2}$ , H <sub>2</sub> CO <sub>3</sub> <sup>*</sup> , CO <sub>3</sub> <sup>2-</sup> , HCO <sub>3</sub> <sup>-</sup> , H <sup>+</sup> , OH <sup>-</sup> , H <sub>2</sub> O
6	Silica species	3	H <sub>4</sub> SiO <sub>4</sub> , H <sub>3</sub> SiO <sub>4</sub> <sup>-</sup> , H <sub>2</sub> SiO <sub>4</sub> <sup>2-</sup>

Kinetic reactions: calcite precipitation/dissolution, dolomite dissolution

Activity coefficients: extended Debye-Hückel equations, Pitzer expressions

86

## Lysimeter Study



87



## HYDRUS Package for Modflow

### The Unsaturated Flow Package for Modflow-2000

Hyeyoung Sophia Seo, Navin Twarakavi, Jirka Šimůnek, and Eileen P. Poeter

Seo, H. S., J. Šimůnek, and E. P. Poeter, Documentation of the HYDRUS Package for MODFLOW-2000, the U.S. Geological Survey Modular Ground-Water Model, *GWMI 2007-01*, International Ground Water Modeling Center, Colorado School of Mines, Golden, Colorado, 96 pp., 2007.

Twarakavi, N. K. C., J. Šimůnek, and H. S. Seo, Evaluating interactions between groundwater and vadose zone using HYDRUS-based flow package for MODFLOW, *Vadose Zone Journal*, doi:10.2136/VZJ2007.0082, Special Issue "Vadose Zone Modeling", 7(2), 757-768, 2008.

91

---

---

---

---

---

---

---

---

---

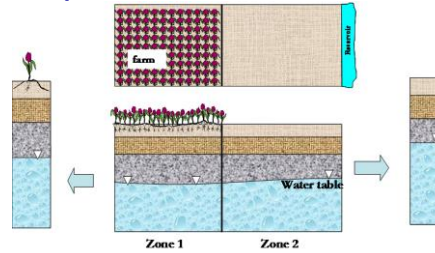
---

---

---

## HYDRUS Package: Zoning

MODFLOW model domain is grouped into zones based on similarities in soil hydraulic characteristics, hydrogeology and meteorology. A HYDRUS vertical profile is assigned to each of the zones on which the 1D Richards equation is used.



92

---

---

---

---

---

---

---

---

---

---

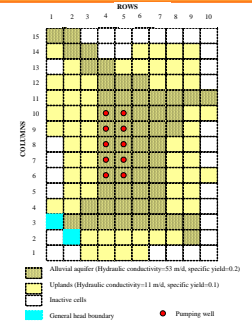
---

---

## HYDRUS - MODFLOW - Case Study

### Hypothetical regional-scale ground water flow problem:

Model domain, spatial distribution of hydraulic conductivities and specific yields, wells (red circles) and general head boundaries.



93

---

---

---

---

---

---

---

---

---

---

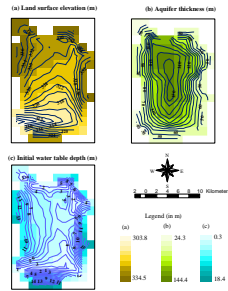
---

---

## HYDRUS - MODFLOW - Case Study

### Hypothetical regional-scale ground water flow problem:

- a) Land surface elevation
- b) depth to bedrock
- c) water table depth at the beginning of the simulation



94

---

---

---

---

---

---

---

---

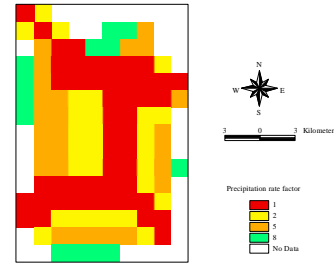
---

---

## HYDRUS - MODFLOW - Case Study

### Hypothetical regional-scale ground water flow problem:

Zonation showing the spatial distribution of precipitation



95

---

---

---

---

---

---

---

---

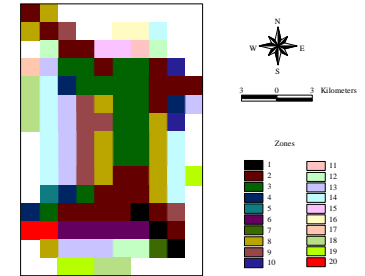
---

---

## HYDRUS - MODFLOW - Case Study

### Hypothetical regional-scale ground water flow problem:

MODFLOW zones used to define HYDRUS soil profiles



96

---

---

---

---

---

---

---

---

---

---

## HYDRUS - MODFLOW - Case Study

**Hypothetical regional-scale ground water flow problem:**

Ground water table fluxes (recharge vs discharge) as predicted by the HYDRUS package at the end of stress periods (a) 3, (b) 6 and (c) 12.

97

---

---

---

---

---

---

---

---

---

---

## New HYDRUS Web Site

[www.pc-progress.com/en/default.aspx](http://www.pc-progress.com/en/default.aspx)

101

---

---

---

---

---

---

---

---

---

---

## New HYDRUS Web Site: Public Libraries

102

---

---

---

---

---

---

---

---

---

---

