Also, we can use the Youngs-La Place equation to related pore radius to soil pressure head, or:

$$\Delta P = \rho_w g h = \frac{2\gamma \cos(\alpha)}{R}$$ Assume $\alpha = 0$

Or, for each pore with radius $R_J = \dfrac{2\gamma}{\rho_w g h_J}$

$$R_J^2 = \frac{4\gamma^2}{(\rho_w g h_J)^2}$$, or

$$K_{sat} = \frac{\rho_w g \Delta\theta_J}{8\eta\tau^2} \frac{4\gamma^2}{(\rho_w g)^2} \sum_{J=1}^{M} \frac{1}{h_J^2}$$

$$K_{sat} = \frac{\Delta\theta_J \gamma^2}{2\eta\tau^2 \rho_w g} \sum_{J=1}^{M} \frac{1}{h_J^2}$$

$$K_{sat} = Constant \sum_{J=1}^{M} \frac{1}{h_J^2}$$

Where all values for Constant are known, except τ !!!!!!!!

Possibly, $\tau = \tau(\theta)$, as $\tau = L_c/L$
HOW ????

$$K_{sat} = Constant \sum_{J=1}^{M} \frac{1}{h_J^2}$$

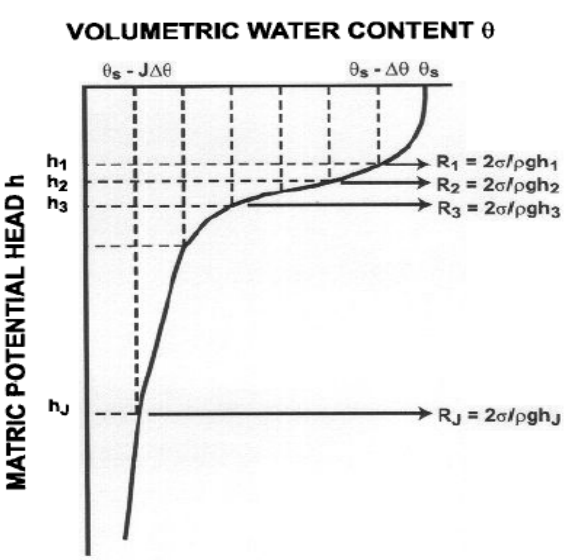
where K_{sat} is controlled by total soil pore space (all pores are filled with water) , hence for all pore classes J =1 (largest pores) to M (smallest pores).

Hence,

$$K(\theta_s - i.\Delta\theta_J) = Constant \sum_{J=i+1}^{M} \frac{1}{h_J^2}$$

i = 0, . . . , (M-1)

with i = 0 representing saturated soil, and i = 1 removing soil pores (capillaries) that drain at h = h_1 (corresponding with r_1, Youngs equation). Therefore, this corresponds with J = 2, M.

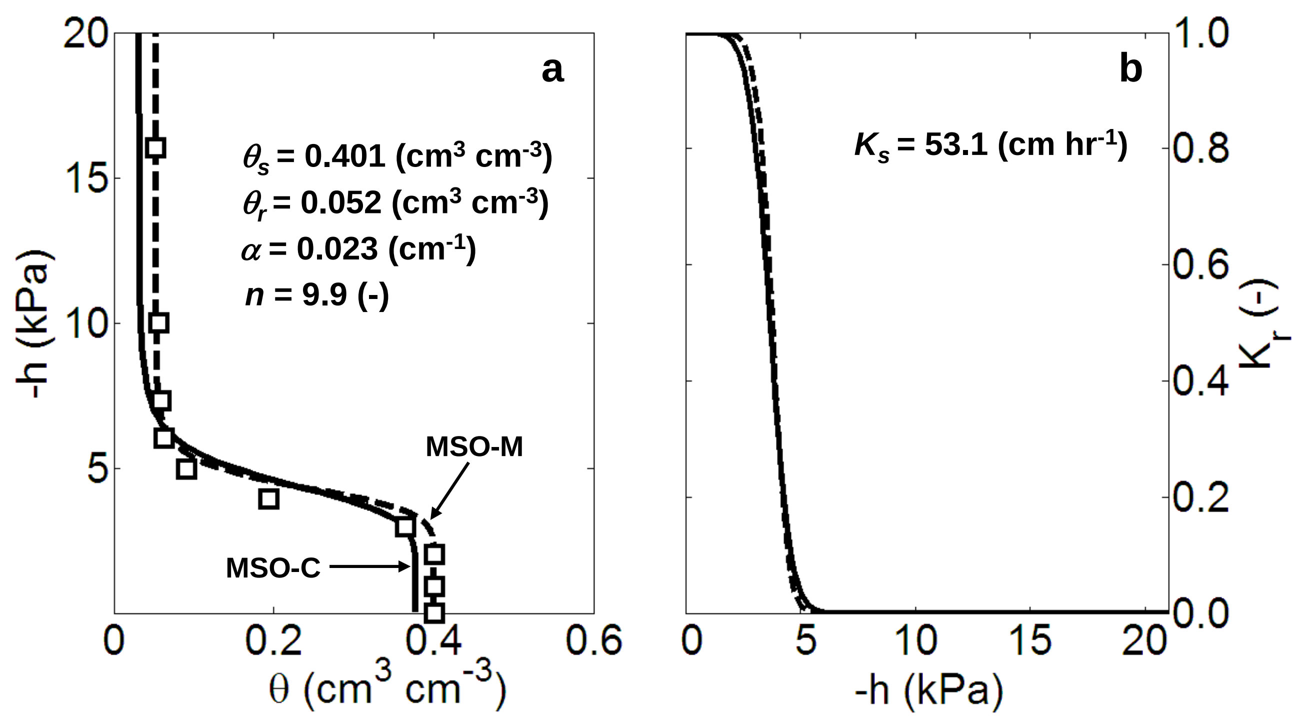


$$R_J = \frac{2\gamma}{\rho_w g h_J}$$

Exercise will ask you (a) to program the solution of unsaturated conductivity data for selected soils (for which soil water retention curves are given), and (2) to determine τ –value by fitting K-solution to known K-data.

Both soil water retention and unsaturated hydraulic conductivity values are determined by van Genuchten-Mualem hydraulic functions:

Van Genuchten retention model   Van Genuchten-Mualem K-model



$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = S_e = (1 + (\alpha\,|h|)^n)^{-m} \qquad K(\theta) = K_s S_e^{0.5}[1 - (1 - S_e^{1/m})^m]^2$$

$$m = 1-(1/n) \text{ and } K_r = K(\theta)/K_{sat}$$

Vadose Zone Hydrology – HYD210	Problem set#2
Due: 04/10/2015	TA: Maziar Kandelous
(Before class starts)	mkandelous@ucdavis.edu

In class we derived an expression to predict the unsaturated hydraulic conductivity from Poiseuille's law.

$$K_s = \frac{\Delta\theta_J \gamma^2}{2\eta\tau^2\rho_w g}\sum_{J=1}^{M}\frac{1}{h_J^2} = Constant_J \sum_{J=1}^{M}\frac{1}{h_J^2} \qquad (1)$$

which leads for the unsaturated K to:

$$K(\theta) = K(\theta_s - i\Delta\theta_J) = Constant_J \sum_{J=i+1}^{M}\frac{1}{h_J^2} \quad i = 0,..............,(M-1) \qquad (2)$$

with i = 0, indicating the contribution to K of all pore classes (K_s), and i=M-1 (J=M) corresponding with K at the lowest water content (only the smallest size pore class contributes).

1. Use the above expressions to predict $K(S_e)$ for the sandy and loamy soils, for which measured retention and unsaturated hydraulic conductivity data were fitted to the van Genuchten-Mualem parameters (use the van Genuchten, vG, retention model to obtain θ for a given h).

   Use $\Delta\theta_J$ = 0.02. List values for all parameters embedded in *Constant*, and their units. Compute τ by matching the predicted K at saturation to the listed K_s value in the table below. Thus, assume that τ is independent of water content, and hence is a constant parameter. So, for this exercise you need to use the vG retention model and Ks value from the table.

Parameter	Sand	Loam
θ_s	0.43	0.43
θ_r	0.045	0.078
α (cm^{-1})	0.145	0.036
n	2.68	1.56
K_s (cm h^{-1})	29.7	1.04
m=1-(1/n)		

Comment [m1]: Jan,
If we use Δθ constant and equal to 0.02, I think here we should say " to obtain h for a given θ".

van Genuchten retention model:

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = S_e = (1 + (\alpha\,|\,h\,|)^n)^{-m} \qquad (3)$$

2. In addition to using the calibrated τ-value at saturation only, approximate the tortuosity dependency on water content, by matching measured with predicted K-data (from the van Genuchten-Mualem K-model) at each of the selected water content values of question 1. Hence, the parameter *Constant* is now variable, and is a function of θ. Fit the newly predicted to the following exponential expression: $\tau = a\, S_e^{\,b}$

van Genuchten-Mualem K-model:

$$K(\theta) = K_s S_e^{0.5}[1 - (1 - S_e^{1/m})^m]^2 \qquad (4)$$