Assignment #4 - Due Thursday, April 27
HYD210 - Hydrologic Modeling of the Vadose Zone

Solve numerically the heat flow equation, using the explicit method

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$ with the boundary conditions:

u(o,t) = u(1,t) = 0

and initial condition

u(x,0) = 2x for 0<x<0.5
u(x,0) = 2(1-x) for 0.5<x<1

a. Solve for u(x,t) for 0<t<1

b. Do this for Δx=0.1, and Δt = 0.001, 0.005 and 0.01.

c. Repeat using Δt = 0.001, and Δx=0.1, 0.05 and 0.02.

d. Compute r for each of the cases. What did you find ?

e. Use graphics to present results.

f. Compare results with analytical solution:

(1) $$u(x,t) = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left[\sin(\frac{n\pi}{2}) \sin(n\pi x) \right] e^{-n^2 \pi^2 t}$$

(Include the analytical solution in the computer program and compare with numerical solution; Discuss).

(2) Alternatively, compare with HYDRUS solution

g. At t=1, the end points are maintained at 1, i.e.,

u(0,t) = u(1,t) = 1 for t>1

whereas the new initial condition is the final solution of (a). Compute the new temperature distribution for 1<t<2.