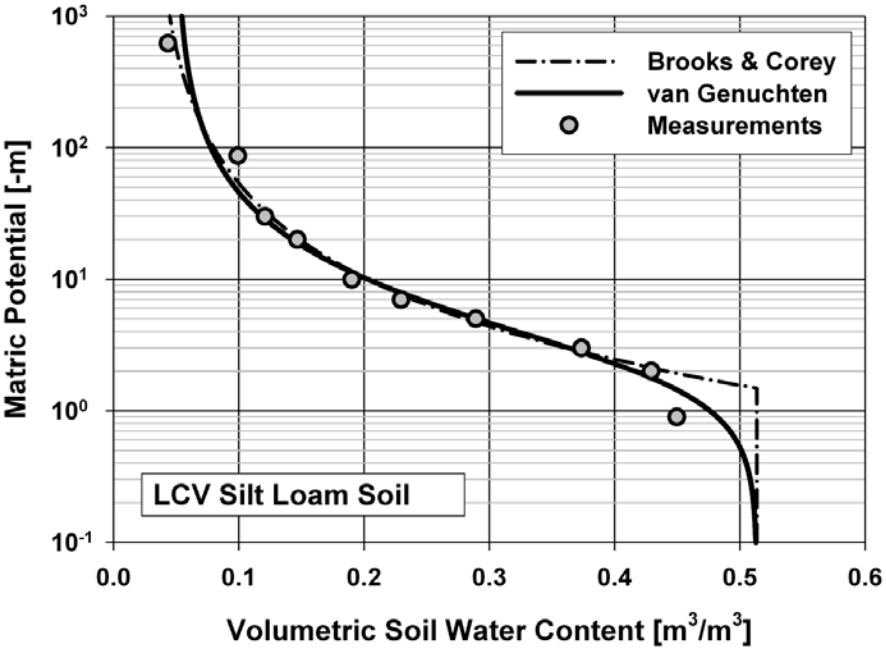
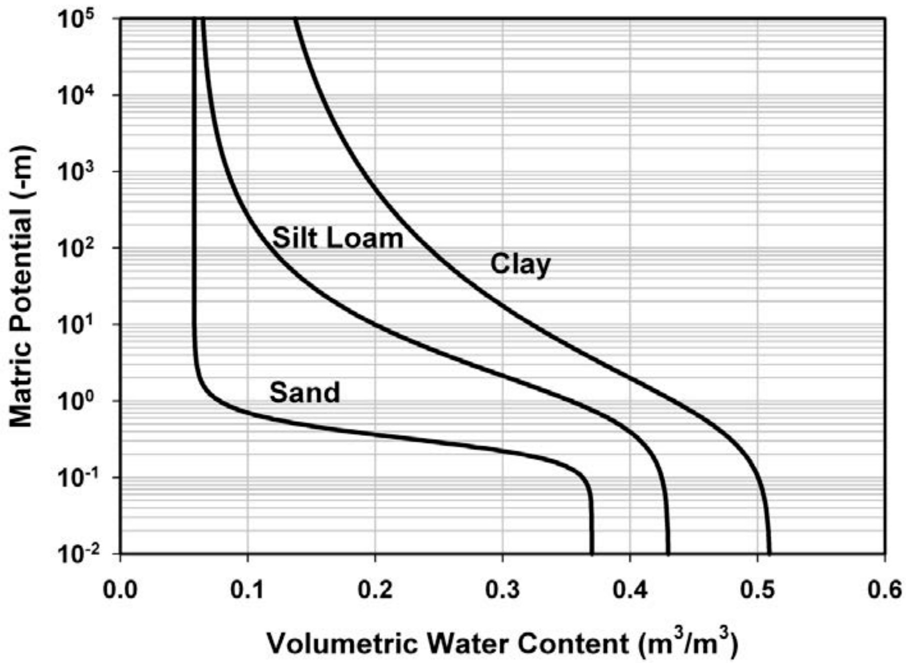
SOIL HYDRAULIC FUNCTIONS

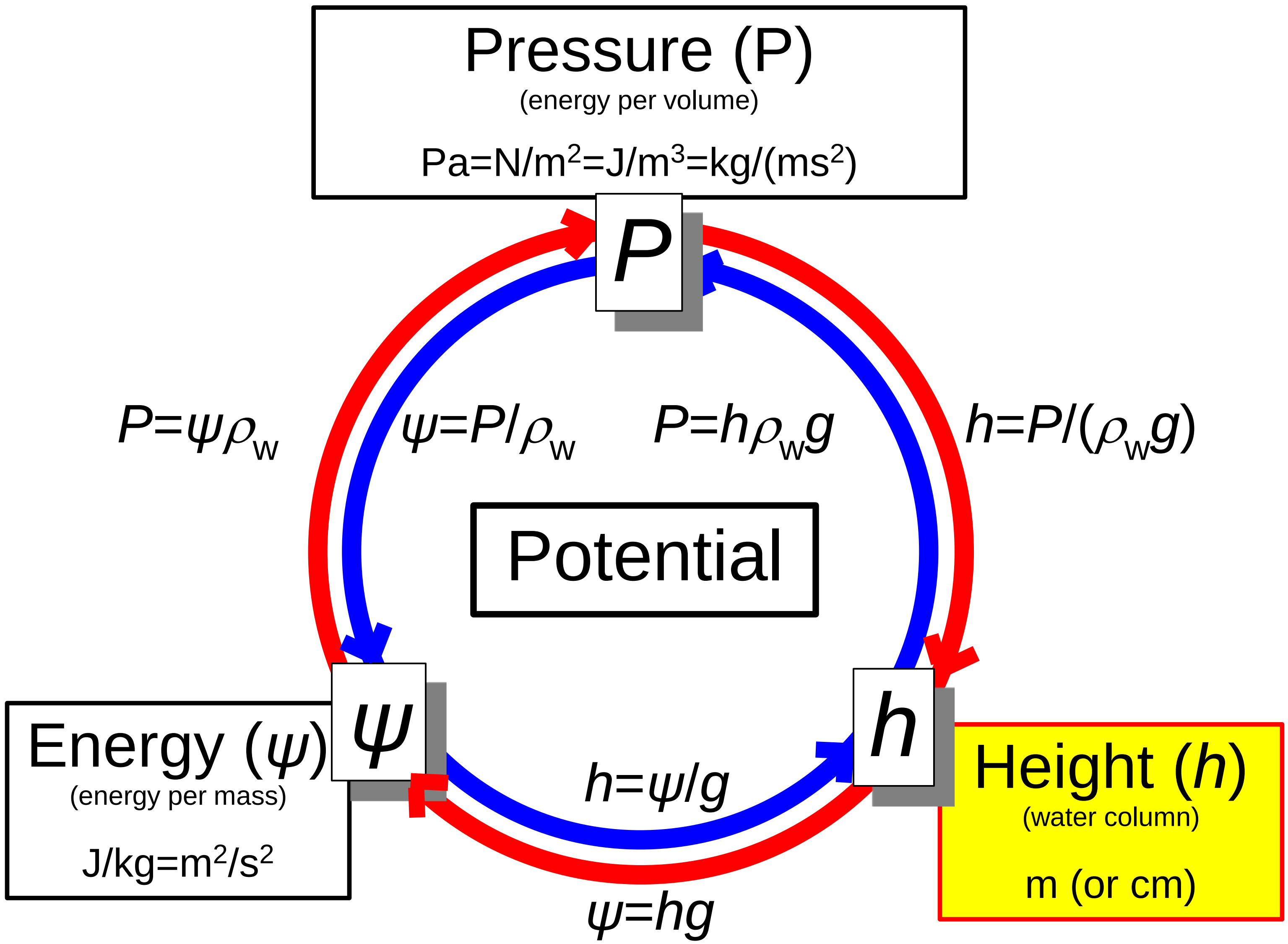
Marcel Schaap
SWES, University of Arizona



Adapted by Jan W Hopmans, UC Davis



Potential



Pressure (P)
(energy per volume)
Pa=N/m²=J/m³=kg/(ms²)
P
P=ψρw
ψ=P/ρw
P=hρwg
h=P/(ρwg)
Potential
Energy (ψ)
(energy per mass)
J/kg=m²/s²
ψ
h
Height (h)
(water column)
m (or cm)
h=ψ/g
ψ=hg

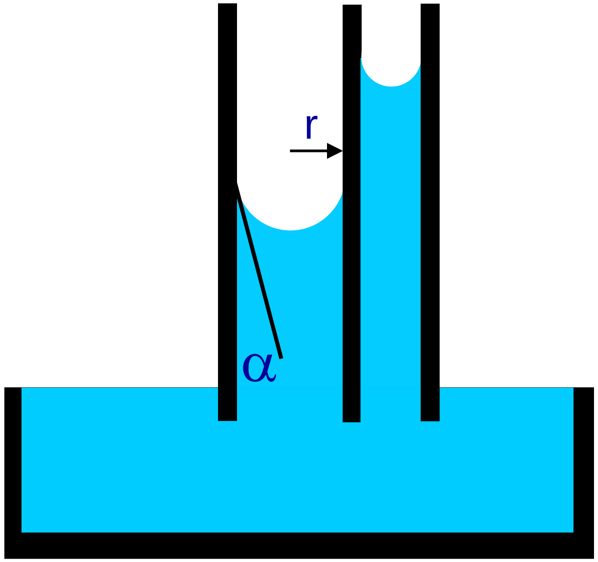
Matric Potential

- Capillary pressure
- Caused by surface tension, σ, of pore water
- Also “van der Waals” forces (films)
- Laplace’s Law:

P	$= 2\sigma\cos\alpha / r$	[Pa]
h	$= 2\sigma\cos\alpha / (\rho_w g r)$	[m]
Ψ	$= 2\sigma\cos\alpha / (\rho_w r)$	[J/kg]

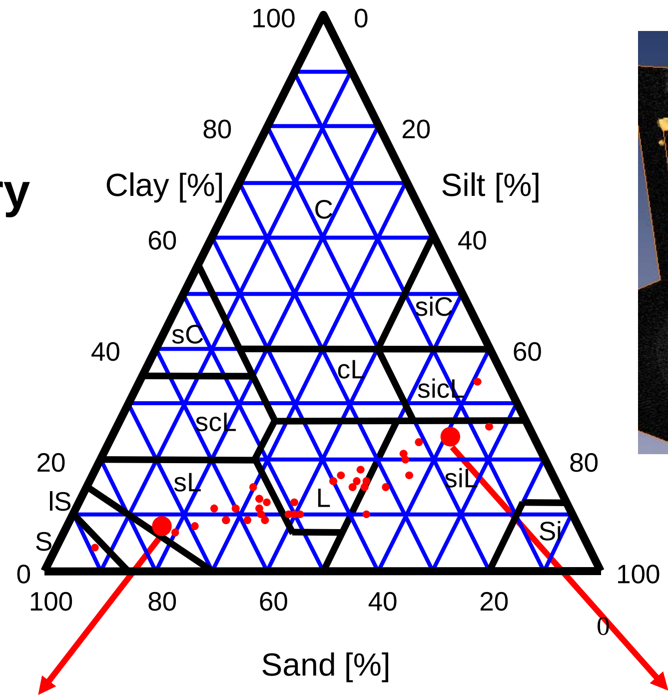
$$r = \frac{2\sigma \cos\alpha}{\rho_w g h}$$

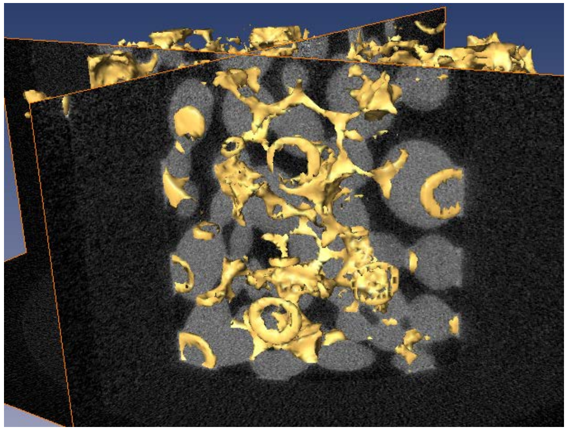
- α=0° for water in most soils
- σ =0.075 N/m for water
- *P* decreases with smaller r
- (beware of the sign for P)



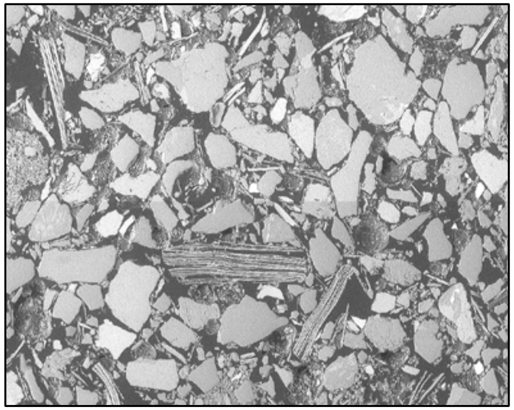
Soils are complex porous media

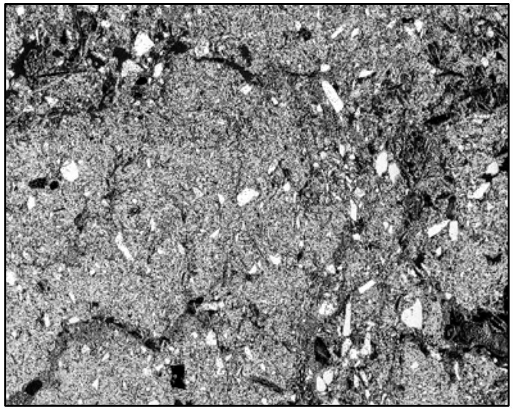
- **Complex solid &**
  Liquid phase geometry
- **Difficult to deal with**
- **Numerical methods**





Wildenschild, OSU





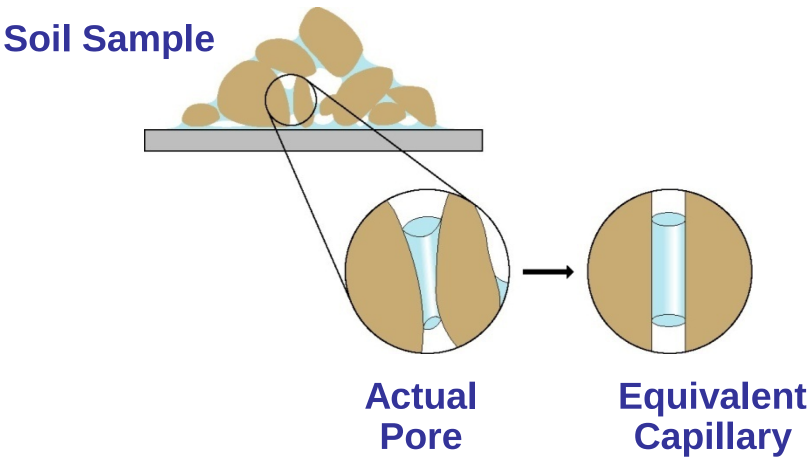
Lebron, Stanford

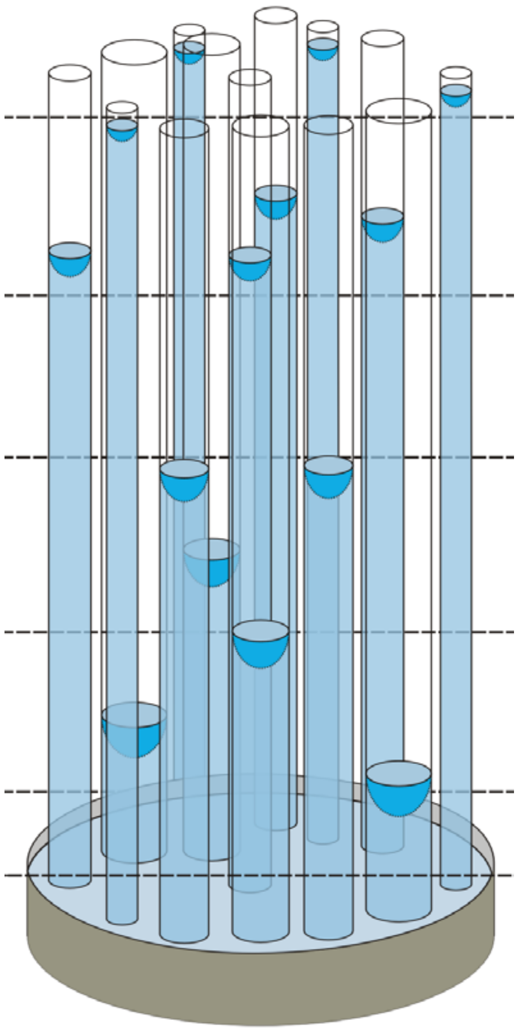
1 mm

Soil pore structure

- Early conceptual models for the liquid distribution in partially saturated porous media are based on the "bundle of cylindrical capillaries" (BCC) representation of pore space geometry (Millington and Quirk, 1961; Mualem, 1976).

- The BCC representation postulates that at a given matric potential a portion of interconnected cylindrical pores is completely liquid filled, whereas larger pores are completely empty.





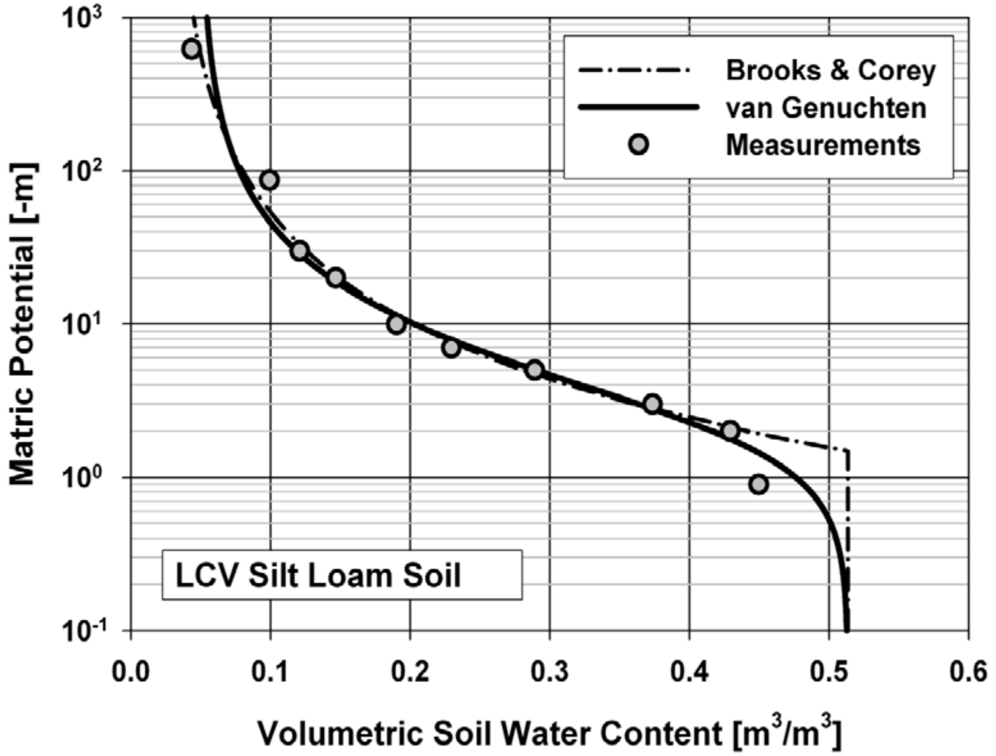
The Soil Water Characteristic Curve (SWC)

- **The Soil Water Characteristic SWC curve describes relationship between soil water content (θ_v or θ) and matric potential (h_m or h) under equilibrium conditions.**

- **The SWC is an important macroscopic soil property that is affected by to pore size distribution and pore interconnectedness, which are strongly affected by texture and structure.**

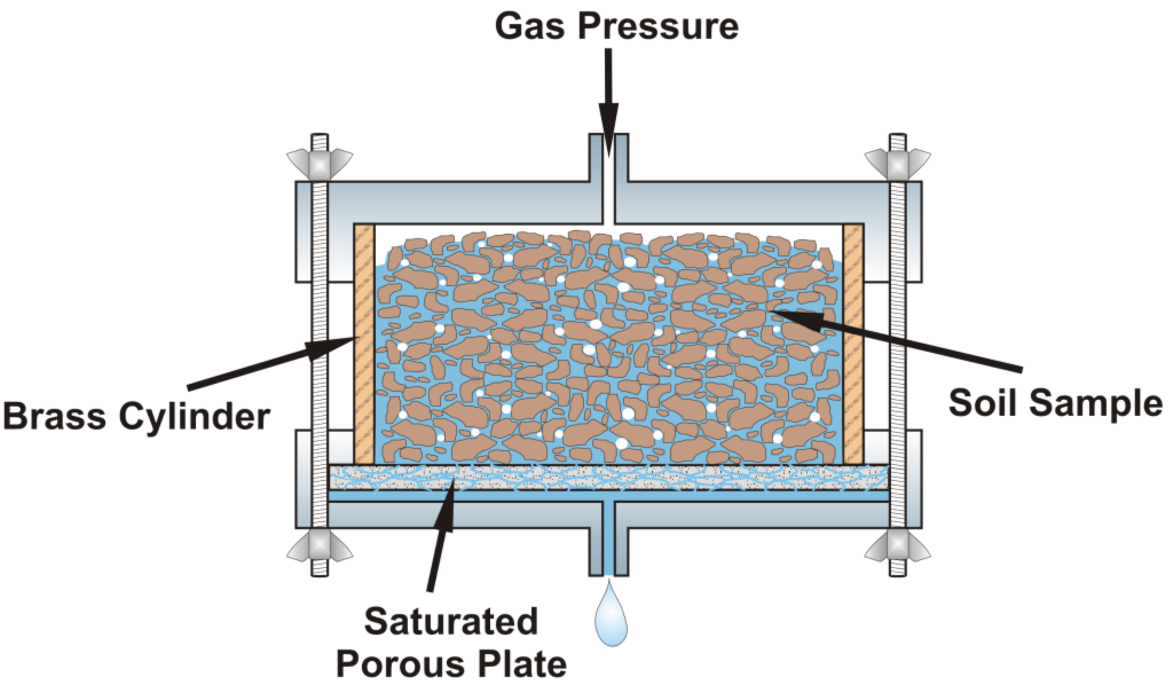
- **The SWC is a primary hydraulic property essential for modeling unsaturated water flow in porous materials.**

- **The SWC function is highly nonlinear, difficult to measure accurately & in-situ, and highly variable in field soils.**



Measurement of SWC: Tempe Cell

The pressure flow cell (Tempe cell) is usually applied for the pressure (matric potential) range from 0 to -10 m.



Close to saturation soil water retention is strongly influenced by soil structure and the natural pore size distribution. Therefore undisturbed core samples are preferred over repacked samples.

Hysteretic Behavior of SWC

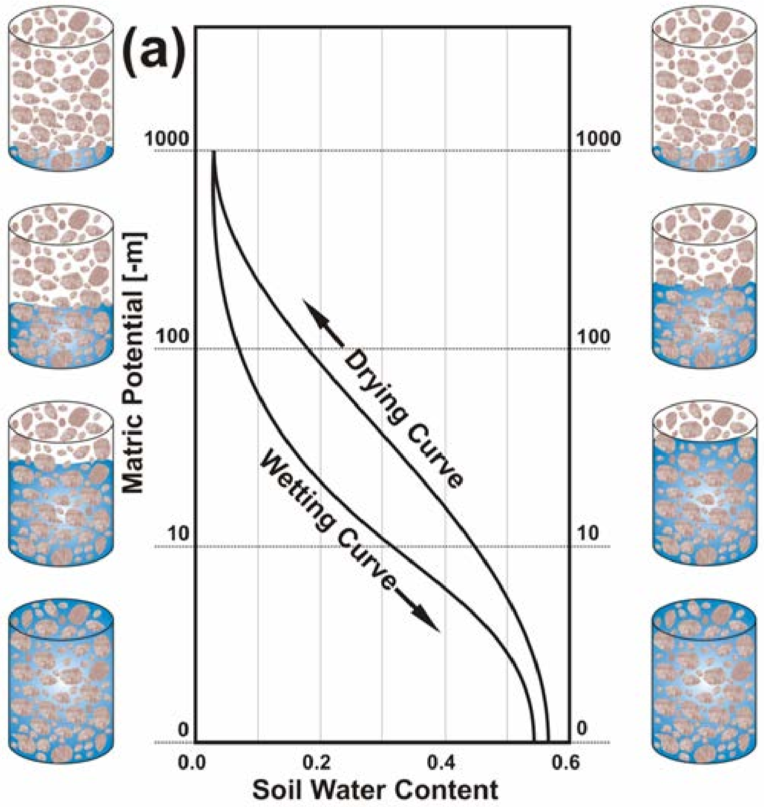
Soil Water Content and **Matric Potential** are not uniquely related and depend on the path of saturation or desaturation.

SWC can be either obtained by desaturation of an initially saturated sample by applying suction or pressure **(DRYING or DRAINAGE CURVE)**, or by gradually wetting of an initially oven-dry sample **(WETTING or IMBIBITION CURVE)**.

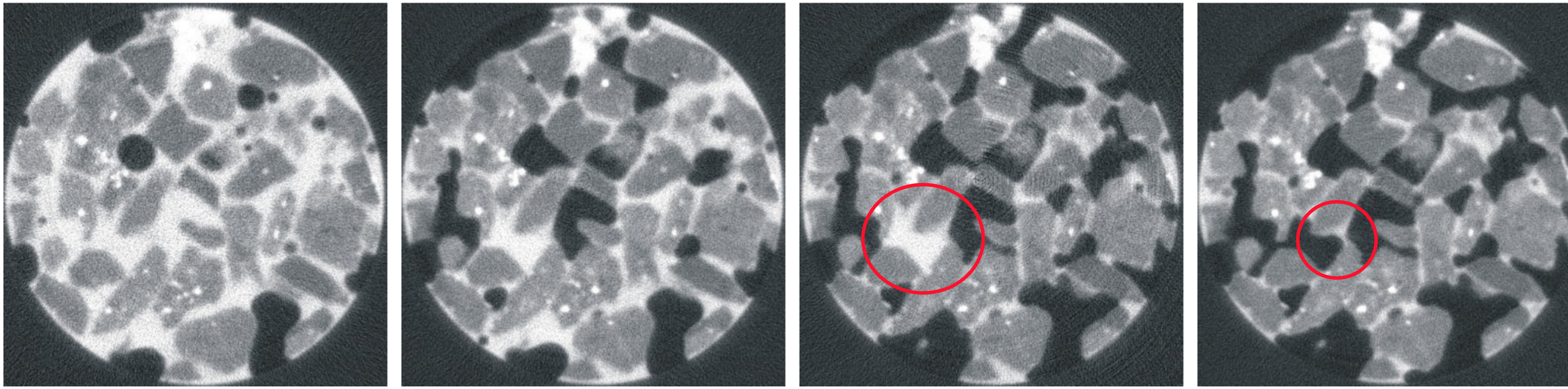
The two procedures often produce different SWC curves - the water content of the drying branch is typically higher than water content of a wetting curve at the same potential.

This phenomenon is known as **HYSTERESIS**

Essentially caused by the pore-scale structure



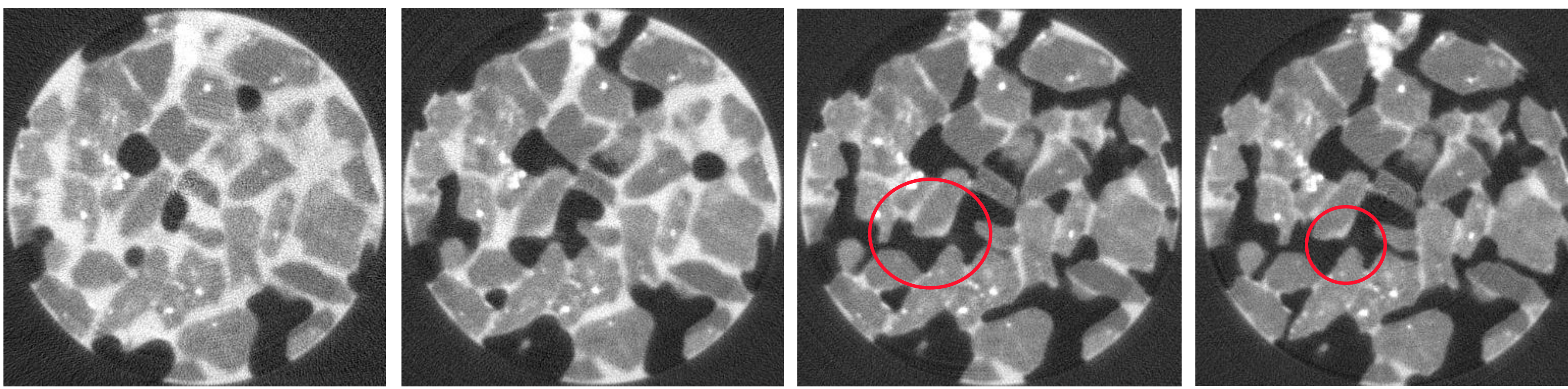
Hysteresis in Microtomography Images
Wildenschild and Hopmans, 2002, J of Hydrology



"saturated"

Source: Wildenschild

drainage →



6 mm diameter (405x406x350 @ 17 μm/pixel) ← **wetting**

Saturated Hydraulic Conductivity

- Transport coefficient for flow through saturated soil.
- Symbol: K_s
- Poiseuille's law for a round tube:

$$Q = \frac{\pi R^4 \Delta P}{8 \eta L}$$

Q:	flow rate m^3/s
R:	tube radius (m)
ΔP:	pressure difference (Pa or kg/[m s^2])
η:	viscosity (Pa s or kg/[m s])
L:	length of the tube

Saturated Hydraulic Conductivity, K_s

- **So, for a bundle of capillary tubes we have to sum over the number of tubes N_i with a particular radius R_i**

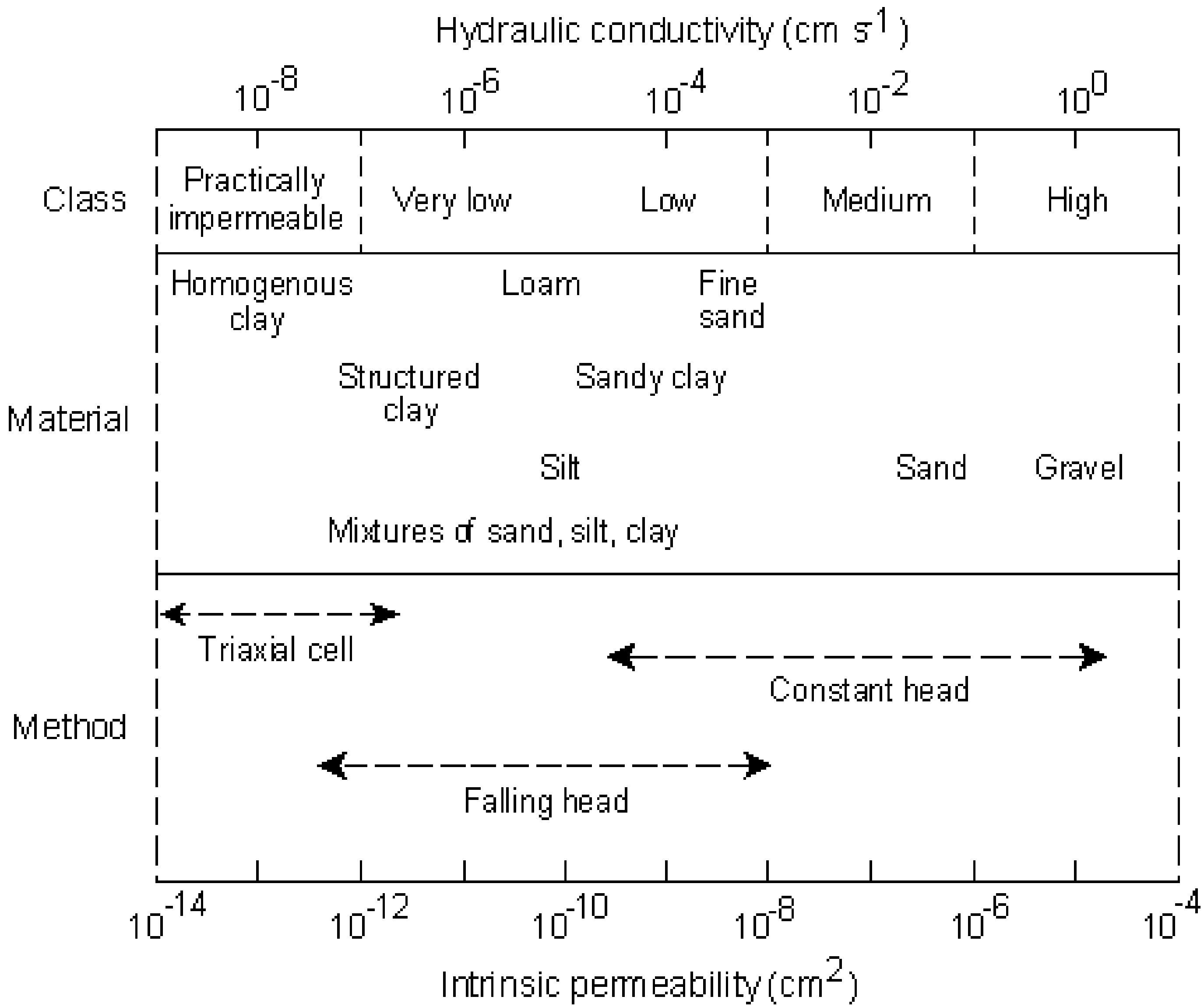
$$Q_t = \sum_i \frac{N_i \pi R_i^4 \Delta P}{8 \eta L} \qquad Q_t = \frac{\pi \Delta P}{8 \eta L} \sum_i N_i R_i^4 \qquad (\mathrm{m^3/s})$$

- **<u>This is a simplification</u>, soils are not bundles of capillaries!**
- **But, it illustrates that larger pores have a dominant contribution.**
- **The transport coefficient of this medium is then (Darcy's Law)**

$$\frac{Q_t L}{A \Delta H} = -K_s \qquad (\mathrm{m/s})$$

- **Where A is the sample area.**

Guidelines for K_s



Unsaturated Hydraulic Conductivity

- largest pores are filled with air (soil water char.)
- Poisseuille's law: Darcy's Law:

$$Q_t = \sum_i \frac{N_i \pi R_i^4 \Delta H}{8 \eta L} \quad (\mathrm{m^3/s})$$

$$\frac{Q_t L}{A \Delta H} = -K_s \quad (\mathrm{m/s})$$

- For a bundle of capillary tubes most of the flow through the largest pores
- Elimination of large pores would lead to a dramatic reduction in K(h).

K(h) data from UNSODA (423 soils)

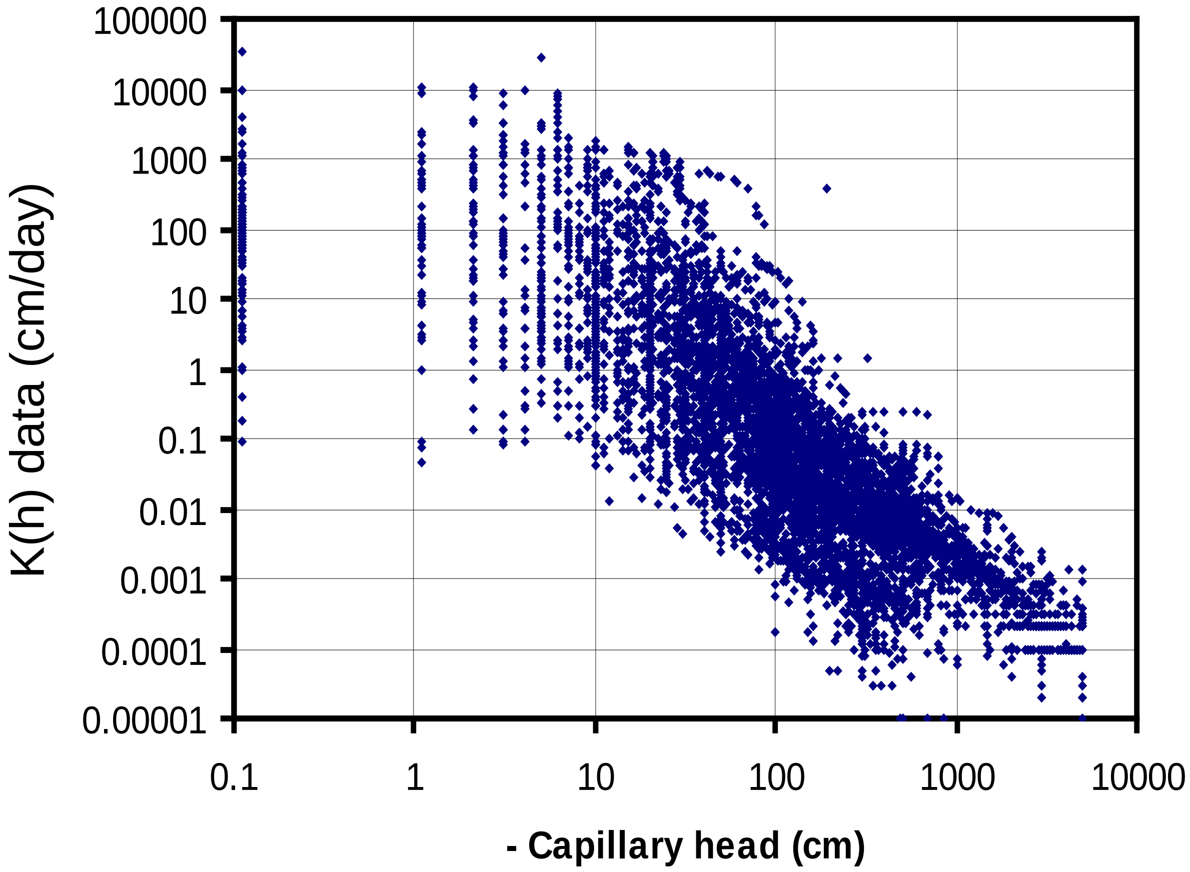
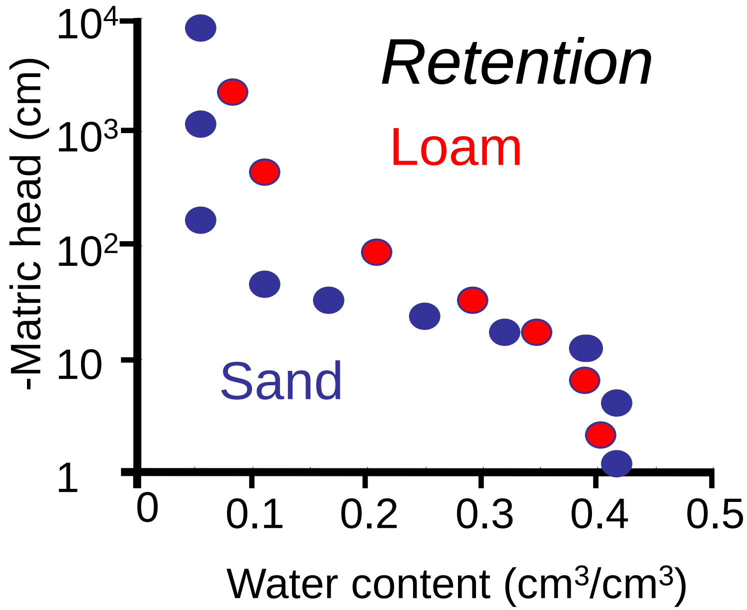
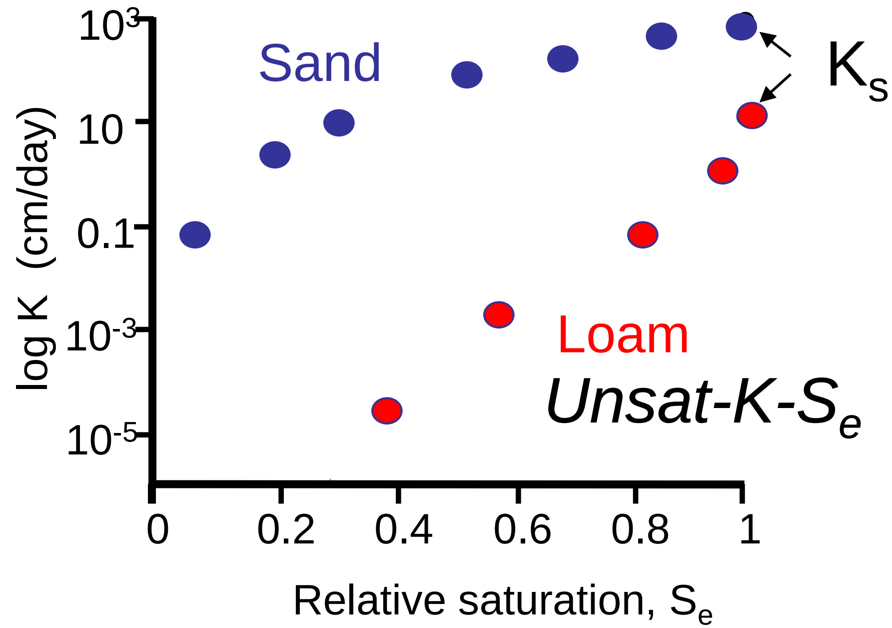
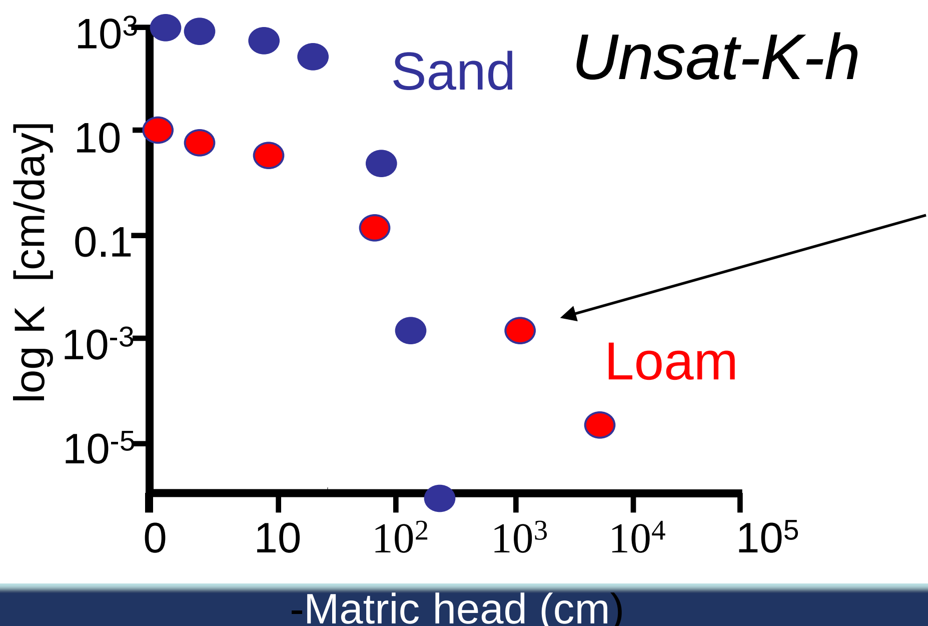


Figure 1. Unsaturated conductivity data versus capillary head for 423 samples of the UNSODA database.

Examples of Hydraulic Characteristics

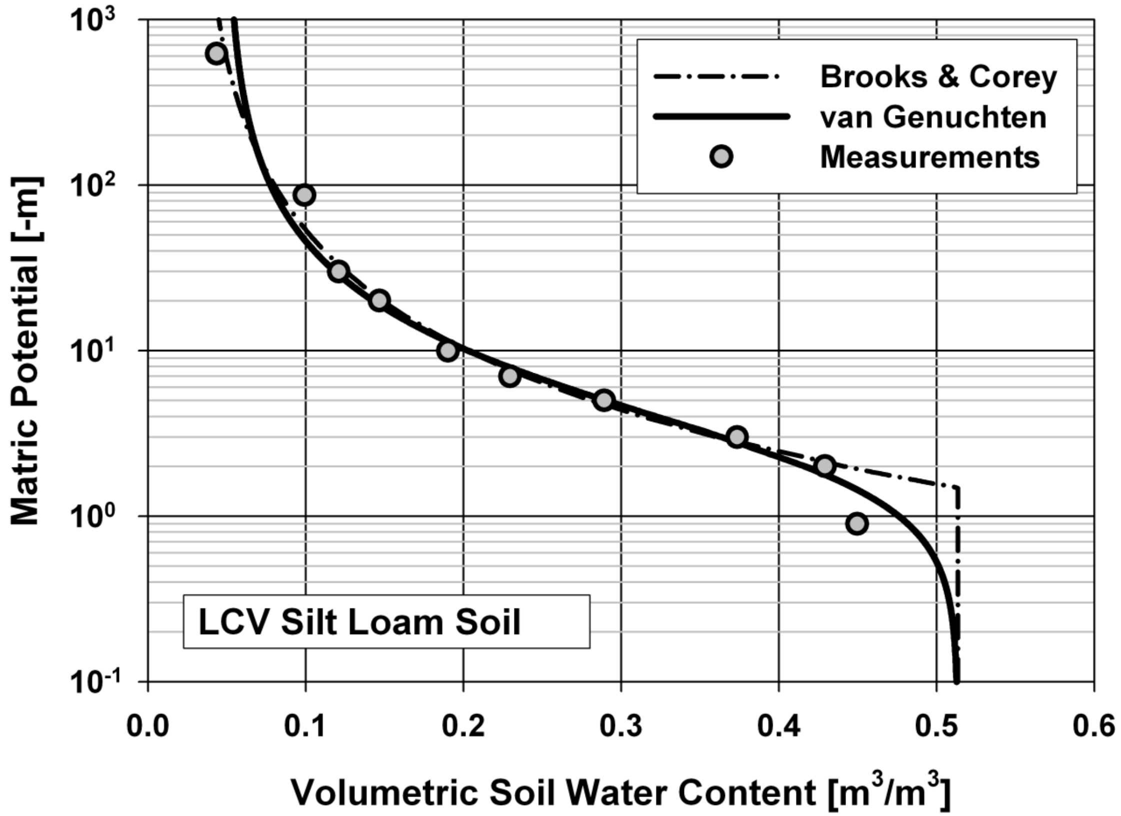






Note that at higher suctions (more negative pressures) the conductivity of loam is <u>higher</u> than that of sand

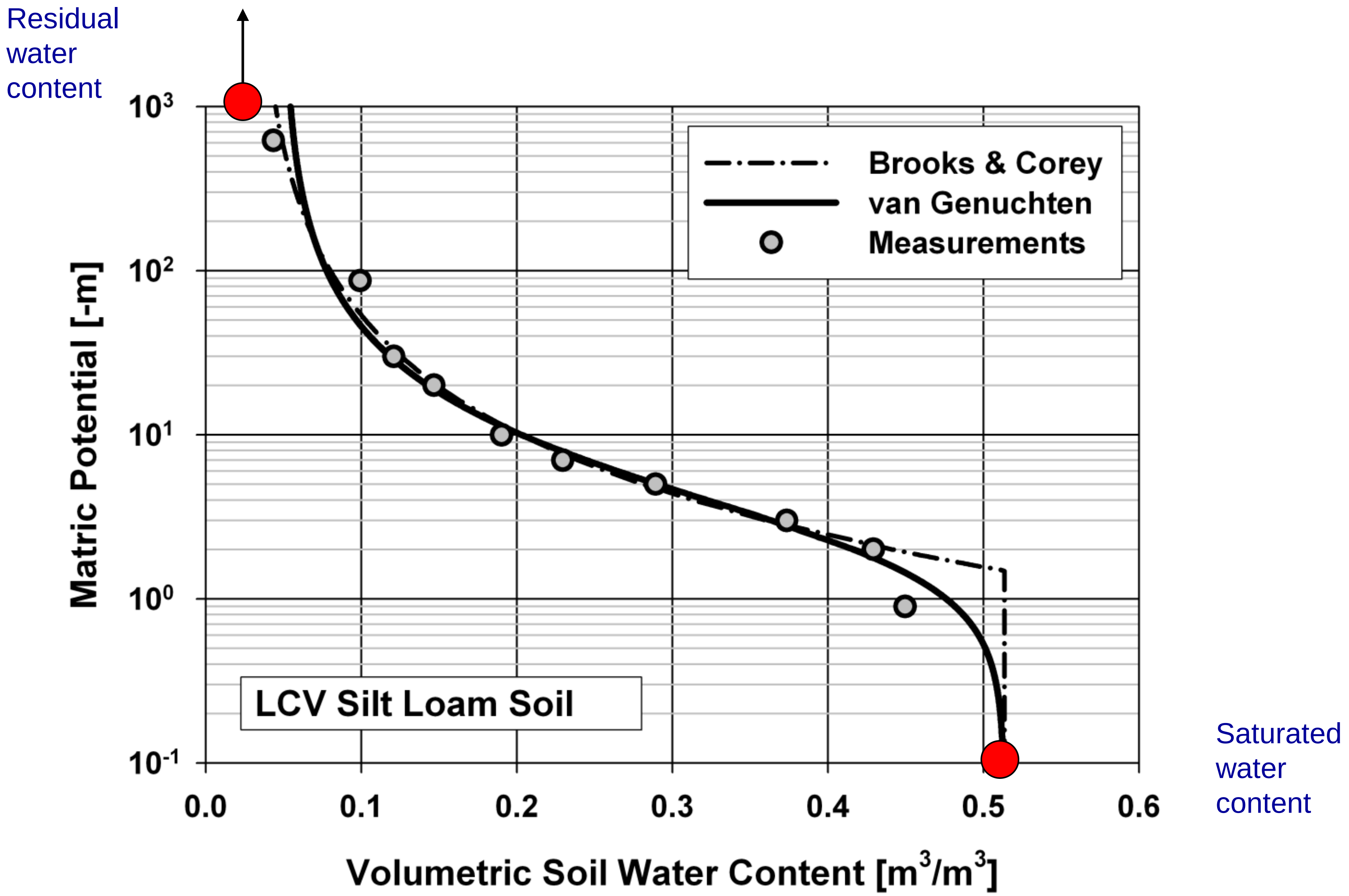
THE SOIL WATER CHARACTERISTIC
Parametric Models



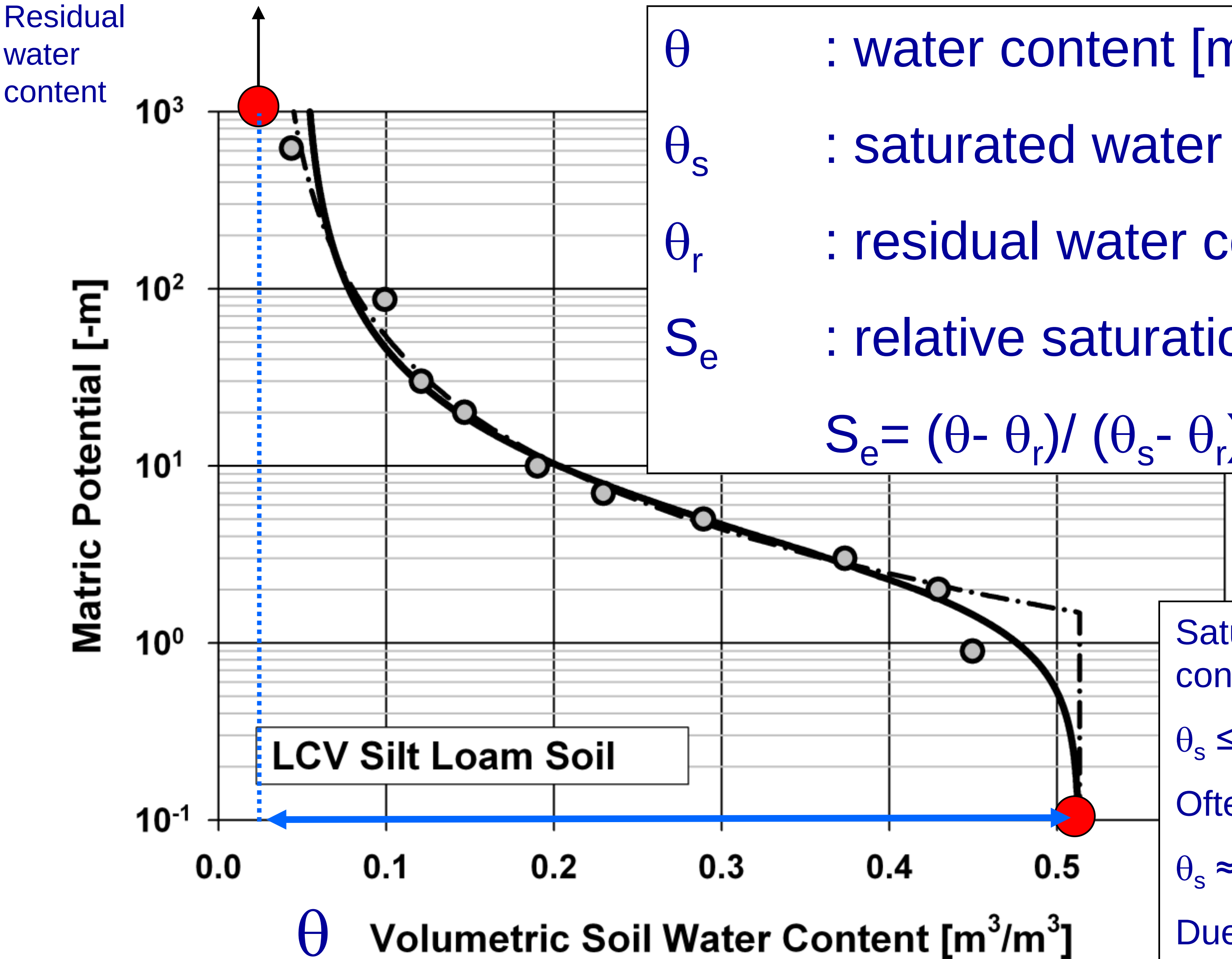
Parametric Models

- **Measuring soil hydraulic properties is laborious and time consuming and expensive. Usually there are only a few data pairs available from measurements.**
- **For modeling and analysis (characterization and comparison of different soils) it is beneficial to represent the hydraulic relationships as <u>continuous parametric functions</u>.**
- **Commonly used parametric models are the <u>van Genuchten</u> and <u>Brooks & Corey</u> relationships. Other models are <u>Kosugi</u> lognormal and <u>Campbell</u> model.**
- **We like to use specific equations rather than splines (or other mathematical ways to describe/interpolate data) because the equation parameters can be interpreted or communicated.**
- **No "true" SWC model defined based on (microscopically) observed pore size distributions.**

Characteristic Points



Characteristic Points



θ : water content [m^3/m^3]

θ_s : saturated water content

θ_r : residual water content

S_e : relative saturation

$S_e = (\theta - \theta_r) / (\theta_s - \theta_r)$ [0..1]

Saturated water content

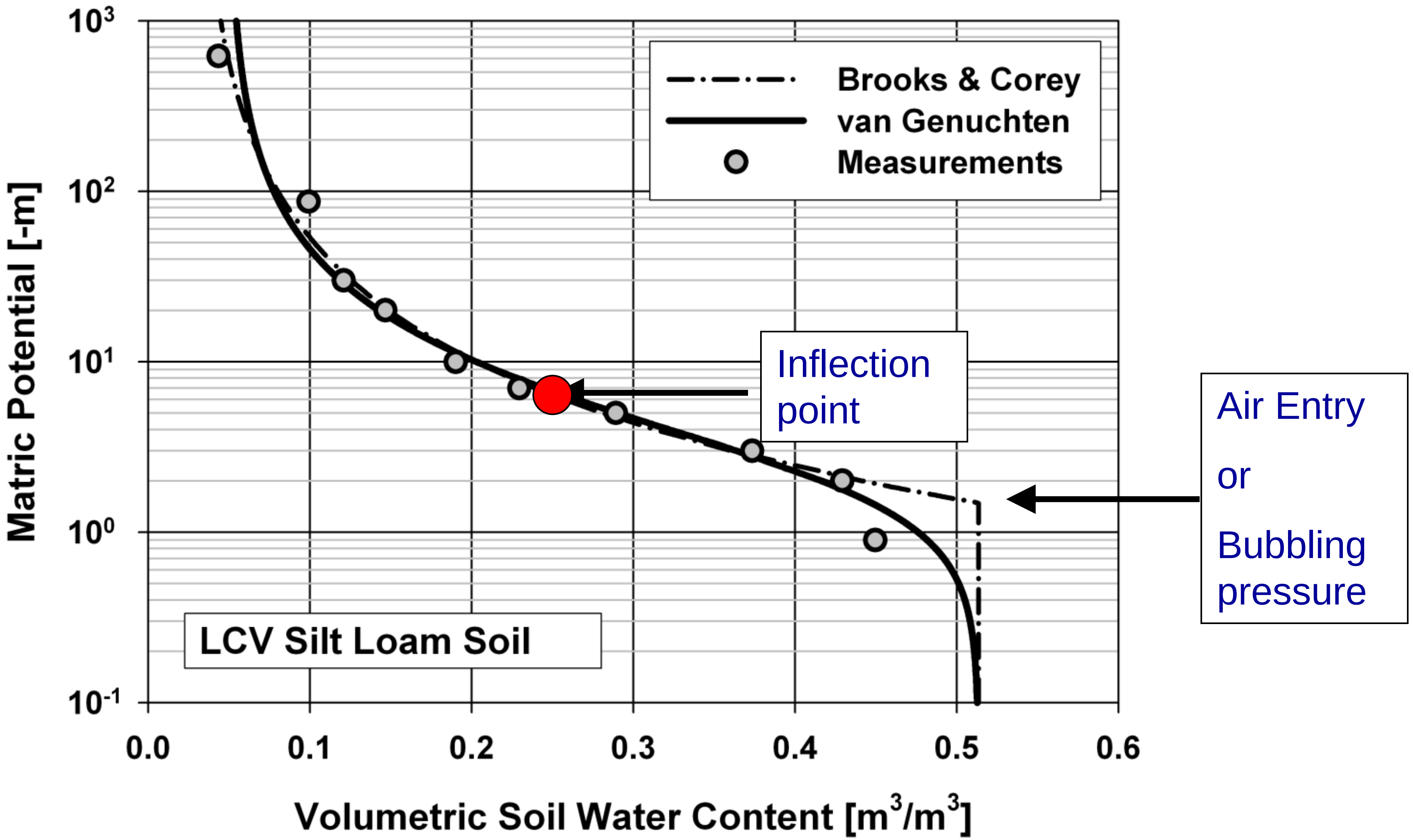
$\theta_s \leq$ porosity (ϕ)

Often:

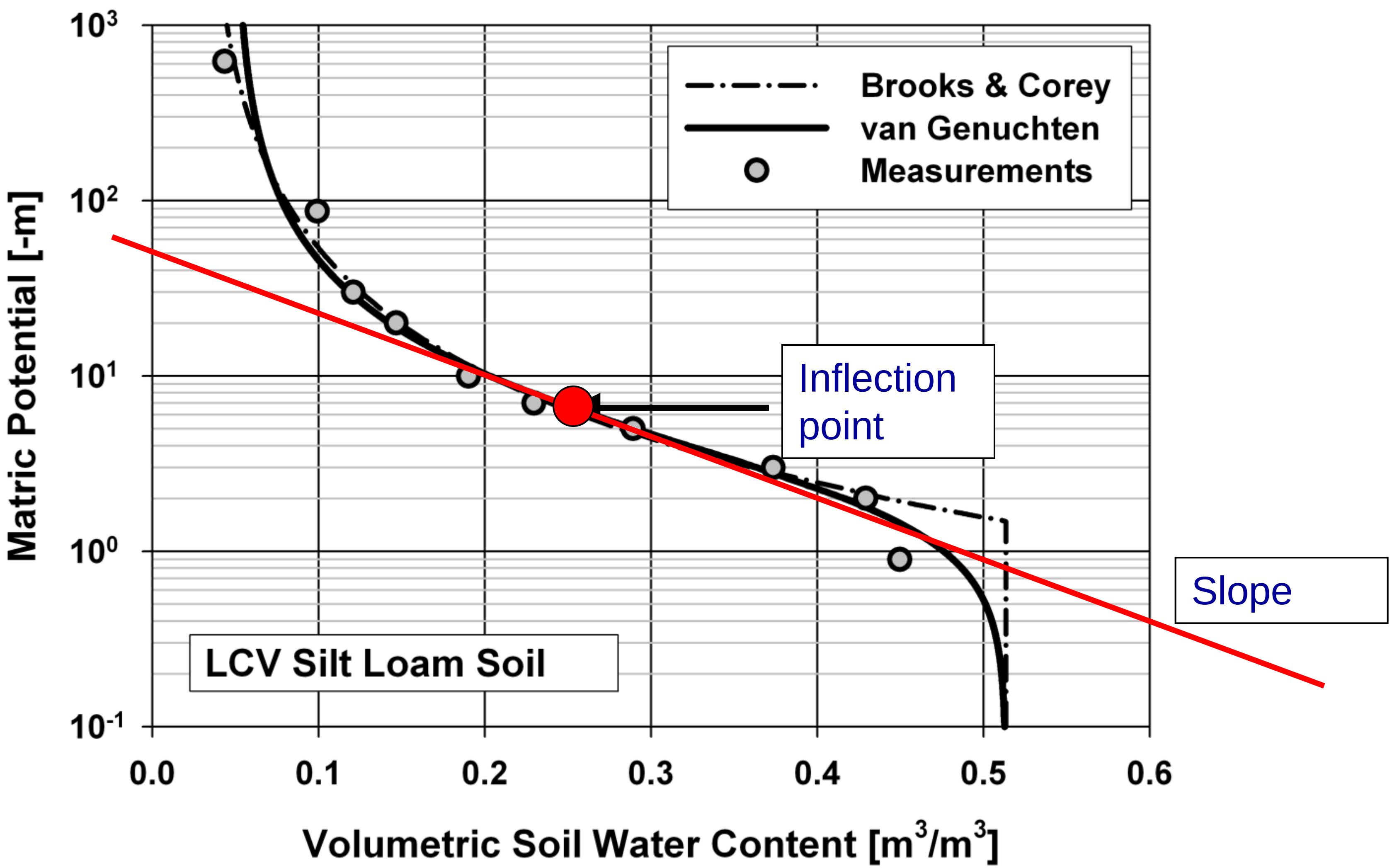
$\theta_s \approx 0.9\ \phi$

Due to air trapping

Van Genuchten and Brooks & Corey Models



Van Genuchten and Brooks & Corey Models



Brooks-Corey | Van Genuchten

Brooks-Corey

4 parameters

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(h/h_b)^{\lambda}} \quad h > h_b$$

$$\theta(h) = \theta_s \quad h \le h_b$$

θ_r residual water content

θ_s saturated water content

h_b air entry pressure

λ pore distribution param.

Van Genuchten

4 or 5 parameters

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{[1+(\alpha h)^n]^m}$$

α inverse air entry pressure

$h_b \approx 1/\alpha$

n pore distribution param.

$\lambda \approx n-1$

m often m=1-1/n

Inflection point (second derivative is zero)

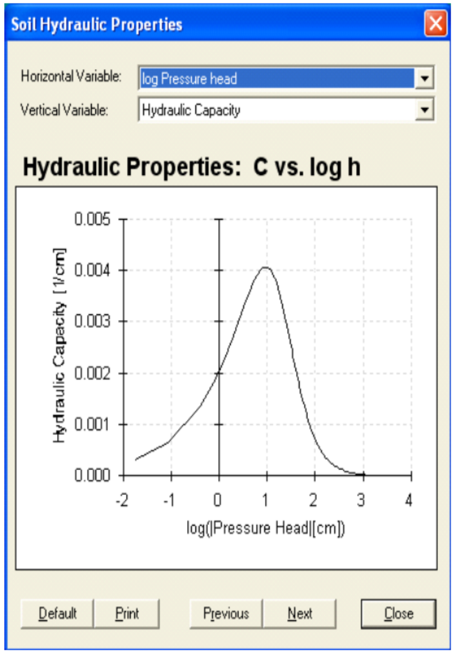


Figure 2.35 Output from the RETC program showing the capacity function for the water retention data for the Ap horizon from Table 2.4.

$$C(h) = \frac{dS_e}{dh} \, has \max imum\ value, or \ \frac{d\left(dS_e / dh \right)}{h} = 0$$

$$\frac{d^2 S_e}{dh^2} = 0, \ at \ h_i$$

$$VanGenuchten : h_i = -\frac{m^{1-m}}{\alpha} = \frac{1}{\alpha} (m \approx 1 \ or \ n \approx \infty)$$

Brooks-Corey | Van Genuchten

Brooks-Corey

4 parameters

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(h/h_b)^{\lambda}} \qquad h < h_b$$

$$\theta(h) = \theta_s \qquad h > h_b$$

- θ_r residual water content
- θ_s saturated water content
- h_b air entry pressure
- λ pore distribution param.

Van Genuchten

4 or 5 parameters

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{[1 + (\alpha h)^{n}]^{m}}$$

- α inverse air entry pressure

  $h_b \approx 1/\alpha$
- n pore distribution param.

  $\lambda \approx n-1$
- m often: $m=1-1/n$

Brooks-Corey / Van Genuchten

Brooks-Corey

4 parameters

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(h/h_b)^{\lambda}}$$ h > h_b

$$\theta(h) = \theta_s$$ h ≤ h_b

Van Genuchten

4 or 5 parameters

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{[1+(\alpha h)^n]^m}$$

If n-value is relatively large:

$$(1+(\alpha h)^n)^m = (\alpha h)^{\lambda}, or,\ \alpha = \frac{1}{h_b}, and$$

$$\lambda = mn = n(1-1/n) = n-1$$

Parametric models

- **The unknown parameters -free parameters- {θ_r, θ_s, h_b, λ} or {θ_r, θ_s, α, n} can be obtained by fitting the models to measured data pairs**
- **There are various computer codes available (e.g., RETC) for estimation of free model parameters. A simple procedure is the application of solver tools that are part of most spreadsheet software packages.**
- **In both models MATRIC POTENTIALS are expressed as positive (absolute) quantities.**

Brooks-Corey

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(|h|/h_b)^{\lambda}}$$

h > h_b

van Genuchten

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{[1+(\alpha|h|)^n]^m}$$

Parametric models

- **The unknown parameters -free parameters- {θ_r, θ_s, h_b, λ} or {θ_r, θ_s, α, n} can be obtained by fitting the models to measured data pairs**
- **There are various computer codes available (e.g., RETC) for estimation of free model parameters. A simple procedure is the application of solver tools that are part of most spreadsheet software packages.**
- **In both models MATRIC POTENTIALS are expressed as positive (absolute) quantities.**

Brooks-Corey

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(|h|/h_b)^\lambda} \qquad h > h_b$$

$$S_e = \left(\frac{h_b}{|h|}\right)^\lambda \qquad h > h_b$$

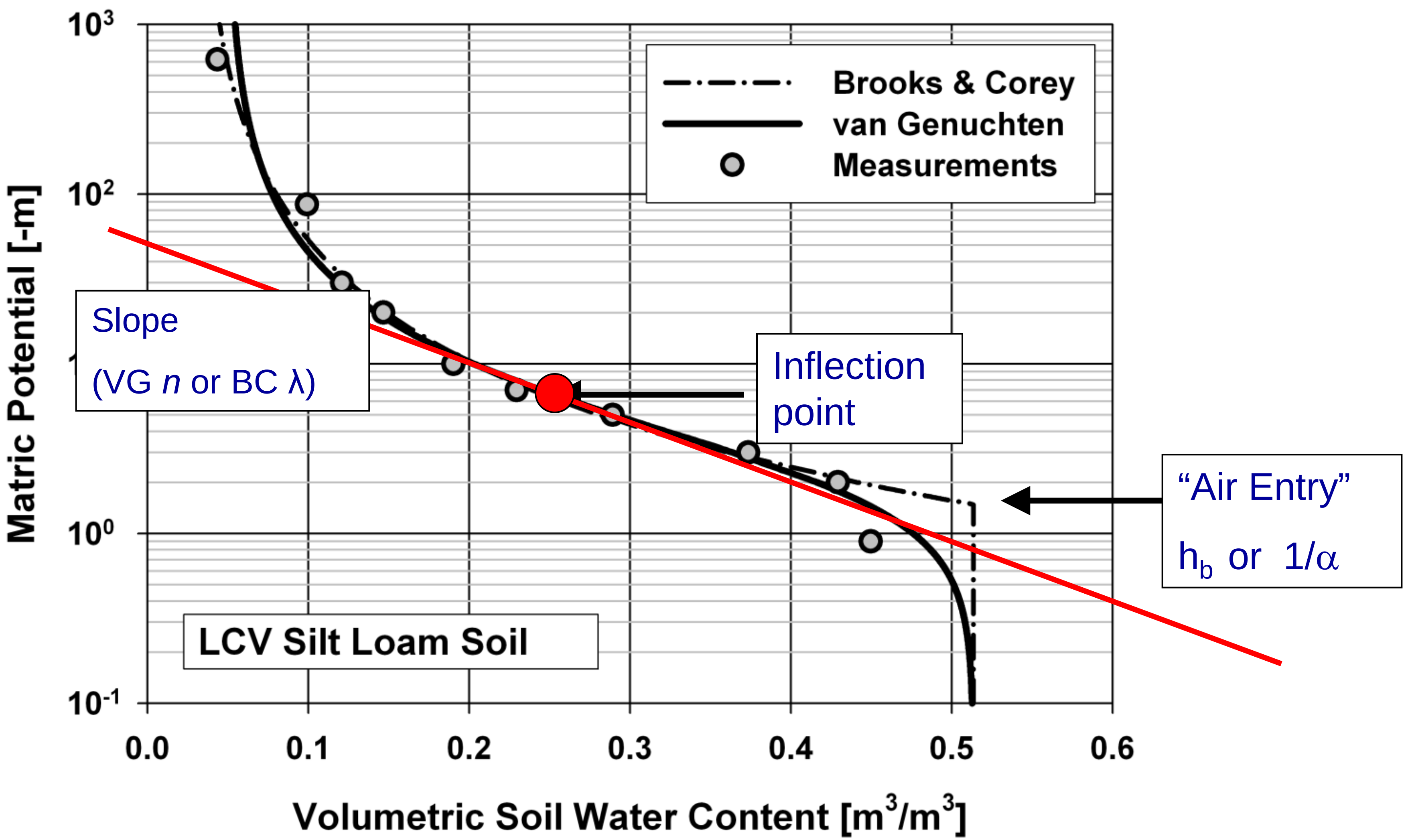
$$S_e = 1 \qquad h \leq h_b$$

van Genuchten

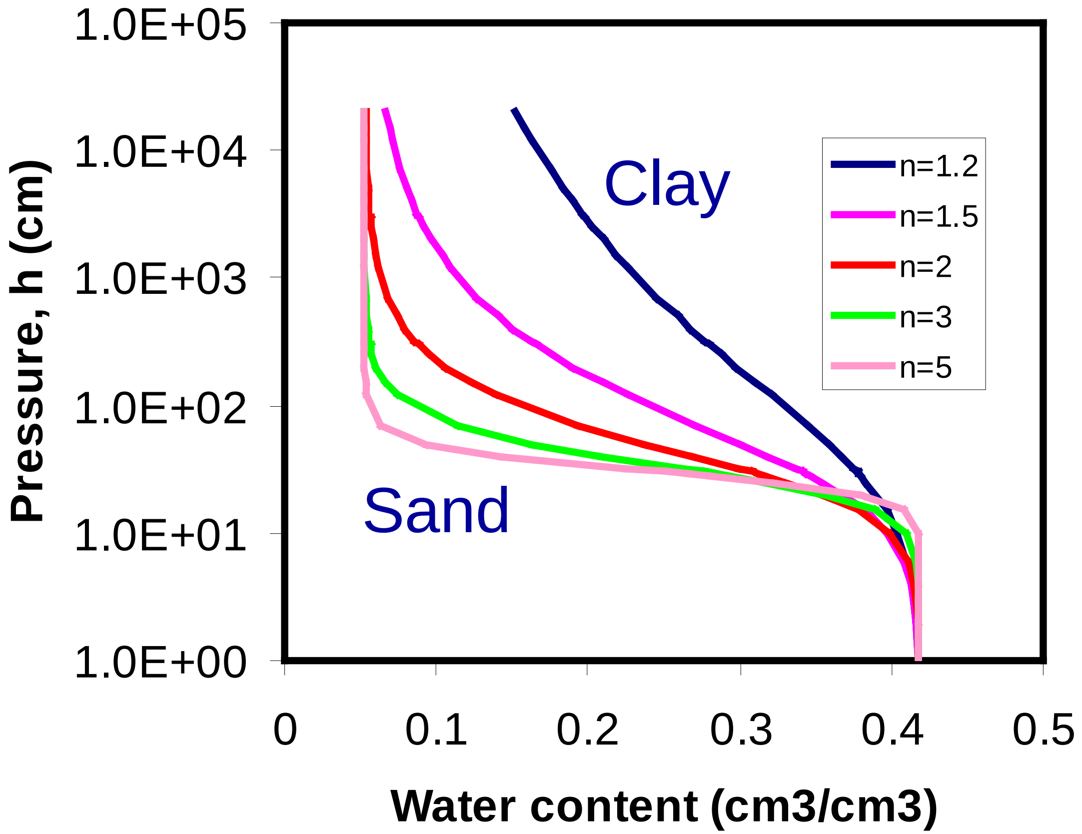
$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{[1 + (\alpha|h|)^n]^m}$$

$$S_e = [1 + (\alpha|h|)^n]^{-m}$$

Van Genuchten and Brooks & Corey Models



Variation in “n” for van Genuchten



λ≈n-1

- Parameter values “capture” the shape of the WRC, facilitating calculations and communication

Campbell (1974)

- **Campbell (1974) is similar to the Brooks-Corey equation, but with θr=0:**

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(|h|/h_b)^{\lambda}} \qquad h > h_b$$

$$\theta(h) = \theta_s \qquad h \le h_b$$

Brooks-Corey

$$\theta(h) = \frac{\theta_s}{(|h|/h_b)^{\lambda}} \qquad h > h_b$$

$$\theta(h) = \theta_s \qquad h \le h_b$$

Campbell

Physically-based models

- All parametric SWC models are empirical: they are successful because they describe observed θ-h data well (for certain soils)
- No "true" SWC model defined based on (microscopically) observed pore size distributions.
- However, it is often observed that soil particle sizes are "log-normally" distributed.
- Assuming that:
  - this implies a log-normal size distribution of pore-volume
  - pores are cylindrical
- We can define what a SWC should look like.
- Arya and Paris (actually distribution-free, not in this lecture)
- Kosugi model

Kosugi's SWC model

- **Skipping over some derivations, the log-normal size distribution of pore volume is:**

$$g(\ln r) = \frac{\theta_s - \theta_r}{\sqrt{2\pi}\sigma r} \exp\left[-\frac{(\ln r - \ln r_m)^2}{2\sigma^2} \right]$$

- **Using capillary equation (Laplace's law with cylindrical pores) :**

$$h = 2\sigma \cos\alpha / \rho_w g r = A/r \quad \text{or} \quad \ln h = \ln A - \ln r$$

$$f(\ln h) = \frac{\theta_s - \theta_r}{\sqrt{2\pi}\sigma h} \exp\left[-\frac{(\ln h - \ln h_m)^2}{2\sigma^2} \right]$$

erfc : complementary error function :

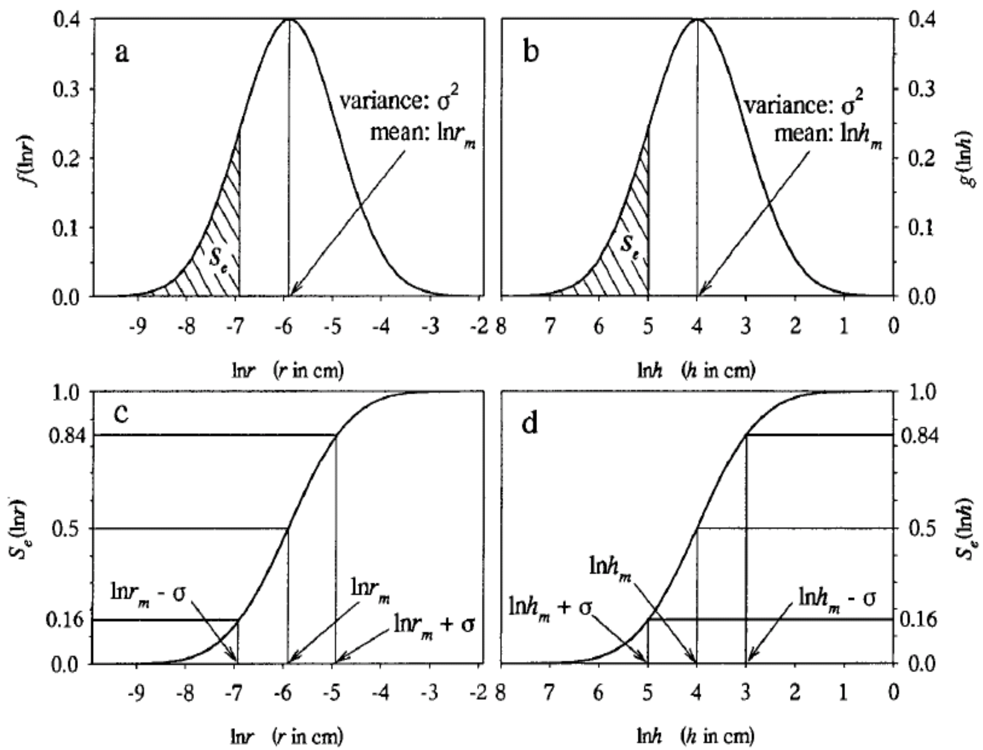
r_m : median pore radius **h_m : median soil water matric head**

σ : standard deviation of log-transformed pore size distribution or matric head distribution

Kosugi's SWC model

This gives us pore volume as a function of the pore radius r. Integration over all pore sizes (and using Laplace's law with cylindrical pores) gives us:

$$S_e(\ln h) = \frac{1}{2}\operatorname{erfc}\left(\frac{\ln h - \ln h_m}{\sqrt{2}\sigma}\right)$$



Functions for Hydraulic Conductivity?

- Empirical models (Gardner, see Hillel p209)
- Pore size distribution models (Mualem, Burdine)

PSD-models are theoretical models that use SWC equations to provide "closed-form" expressions for K(h), or $K(S_e)$. See van Genuchten (1980), Mualem and Dagan(1978), and Raats (1992).

- SWC is used to provide the pore-size distribution.
- pore are filled with water, depending on their size and matric potential and only water filled pores contribute to hydraulic conductivity
- pores of different sizes are connected to each other

Pore Size Distribution Models

$$K_r = \frac{K(S_e)}{K_s}$$

- Mualem

$$K(S_e) = K_s S_e^{\,l} \left[\frac{\int_0^{S_e} |h|^{-1} dS_e}{\int_0^{1} |h|^{-1} dS_e} \right]^2$$

- Burdine

$$K(S_e) = K_s S_e^{\,l} \left[\frac{\int_0^{S_e} |h|^{-2} dS_e}{\int_0^{1} |h|^{-2} dS_e} \right]^1$$

$$K(S_e) = K_s S_e^{\,l} \left[\frac{\int_0^{S_e} |h|^{-\eta} dS_e}{\int_0^{1} |h|^{-\eta} dS_e} \right]^\gamma$$

General form

Mualem-van Genuchten

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{[1+(\alpha h)^n]^m}$$

van Genuchten

and

$$K(S_e) = K_s S_e^{\,l} \left[\frac{\int_0^{S_e} |h|^{-1} dS_e}{\int_0^{1} |h|^{-1} dS_e} \right]^2$$

Mualem

gives

$$K(S_e) = K_s S_e^{L} \{1 - [1 - S_e^{n/(n-1)}]^{1-1/n}\}^2$$

$$S_e = \frac{\theta(h) - \theta_r}{\theta_s - \theta_r} = \frac{1}{[1+(\alpha h)^n]^m}$$

m=1-1/n

Mualem-van Genuchten

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{[1+(\alpha h)^n]^m}$$

and

$$K(S_e) = K_s S_e^{\,l} \left[\frac{\int_0^{S_e} |h|^{-1} dS_e}{\int_0^1 |h|^{-1} dS_e} \right]^2$$

Saturated Hydraulic Conductivity

Tortuosity/pore connectivity (“0.5” or “-1”?)

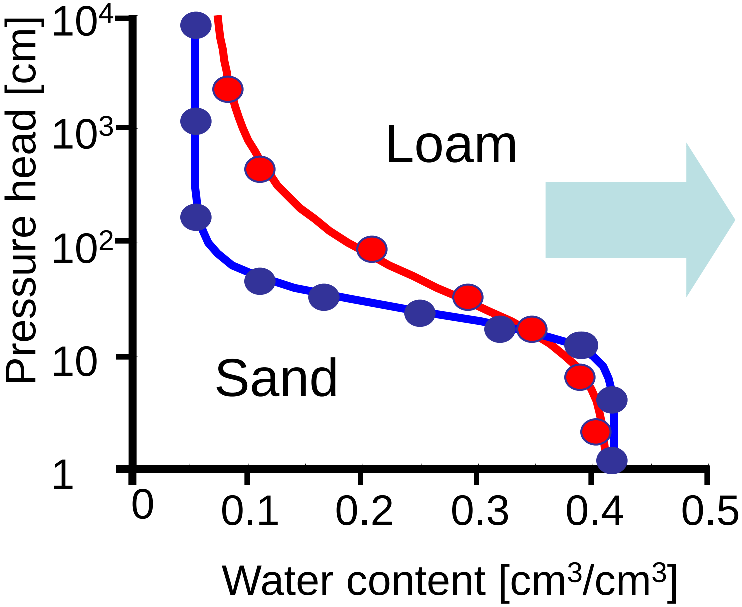
Pore-size distribution

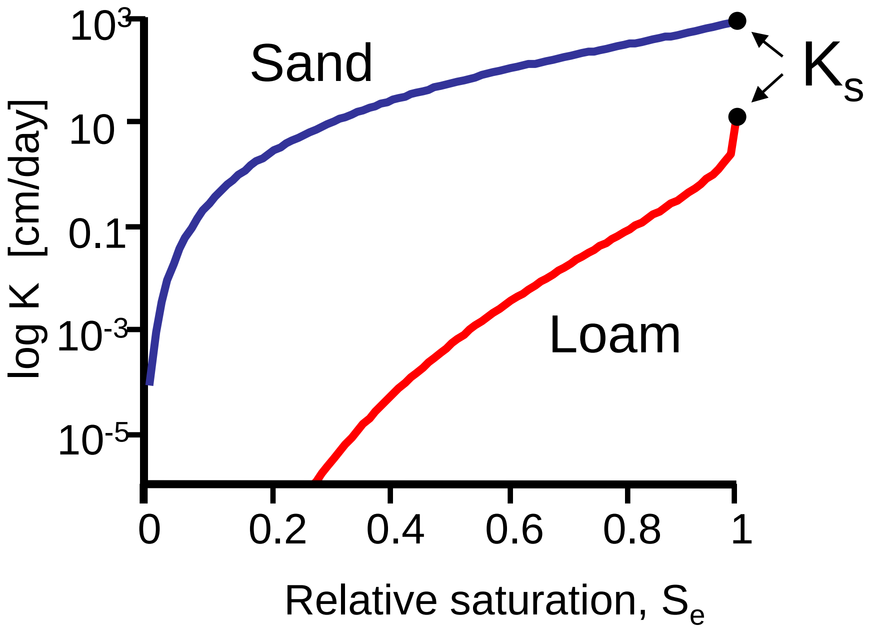
$$K(S_e) = K_s S_e^{L} \{1-[1-S_e^{n/(n-1)}]^{1-1/n}\}^2$$

$$S_e = \frac{\theta(h) - \theta_r}{\theta_s - \theta_r} = \frac{1}{[1+(\alpha h)^n]^m}$$

m=1-1/n

Using MVG to predict Unsat-K





1) Measure K_s

2) Measure WRC

3) Fit VG parameters to WRC

4) Apply MVG to get $K(h, S_e)$

Mualem/Burdine-Brooks Corey

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(h/h_b)^{\lambda}} \qquad h > h_b$$

$$S_e = \left(\frac{h_b}{h}\right)^{\lambda} \qquad h > h_b$$

$$\theta(h) = \theta_s \qquad h \le h_b$$

$$S_e = 1 \qquad h \le h_b$$

$$K(S_e) = K_s S_e^{\,l} \left[\frac{\int_0^{S_e} |h|^{-1} dS_e}{\int_0^1 |h|^{-1} dS_e} \right]^2$$

Mualem

gives

$$K(S_e) = K_s S_e^{\,l+2+2/\lambda}$$

$$K(S_e) = K_s S_e^{\,l} \left[\frac{\int_0^{S_e} |h|^{-2} dS_e}{\int_0^1 |h|^{-2} dS_e} \right]^1$$

Burdine

gives

$$K(S_e) = K_s S_e^{\,l+1+2/\lambda}$$

Mualem/Burdine - Kosugi

- **Kosugi's model (no air-entry parameter)**

$$S_e(\ln h) = \frac{1}{2}\operatorname{erfc}\left(\frac{\ln h - \ln h_m}{\sqrt{2}\sigma}\right)$$

$$Q(x) = [1/(2\pi)]\int_x^\infty \exp-(u^2/2)\,du$$

$$\operatorname{erfc} x = 1 - \operatorname{erf} x = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}\,du$$

Mualem

$$K(S_e) = K_s S_e^{\,l}\left\{Q\left[Q^{-1}(S_e) + \sigma\right]\right\}^2$$

Burdine

$$K(S_e) = K_s S_e^{\,l} Q\left[Q^{-1}(S_e) + 2\sigma\right]$$

- **See Table 3.3.4-2 in "Parametric_models.pdf" for more information (Kosugi, Hopmans and Dane, Methods of Soil Analysis, 2002)"**

Estimating Hydraulic Functions
Pedotransfer Functions (PTF)