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# Estimation of Vadose Zone Water Flux from Multi-Functional Heat Pulse Probe Measurements

Y. Mori,* J. W. Hopmans, A. P. Mortensen, and G. J. Kluitenberg


## ABSTRACT

A small multi-functional heat pulse probe (MFHPP) was applied to further develop measurement methodologies to improve on water flux estimations for unsaturated soils. The temperature responses of four thermistors surrounding a central heater in a 2.7-cm diam. probe were analyzed by the heat transport equation to estimate thermal properties and convective heat flow. Volumetric heat capacity, water content, and thermal diffusivity were estimated from the horizontally placed thermistors, neglecting the convective flow effects in the transverse direction, whereas the water flux density was estimated from the temperature responses to the vertically placed thermistors. A parameter optimization technique was employed to fit the most likely parameters to the relevant analytical solutions. Falling head and multistep outflow experiments yielded independently obtained water flux measurements. Results showed that the estimated volumetric water content corresponded well with independent gravimetric measurements with a RMSE of 0.0056 $m^3$ $m^{-3}$, across a wide range of water fluxes smaller than 0.5 m $d^{-1}$. Thermal diffusivity values as obtained with the MFHHP also agreed well with independently measured thermal diffusivity values, for water flux density values smaller than 2 m $d^{-1}$. For saturated conditions, the estimated water fluxes from the MFHPP measurements were accurate in the range between 0.056 and 27.0 m $d^{-1}$, with a $R$ of 0.995 and RSME of 0.0952 log(m $d^{-1}$) (0.52 m $d^{-1}$). For unsaturated flow, MFHHP estimations significantly overestimated water flux density for flux values smaller than 0.10 m $d^{-1}$. Within these limitations, we conclude that MFHPP methodologies are now available, making possible simultaneous estimation of thermal diffusivity and water flux density in unsaturated soils.


Soil water flux measurements are essential for a better understanding of transport phenomena in the vadose zone and improvement of water and nutrient management practices. In general, water flux estimates are highly uncertain as they are mostly based on Darcy's law, requiring in situ point measurements of hydraulic head gradient and the unsaturated hydraulic conductivity function. Soil water matric potential measurements may include significant measurement errors that preclude accurate hydraulic gradient estimation, especially if gradients are small (Flűhler et al., 1976). In situ physical techniques to estimate the unsaturated hydraulic conductivity are few and difficult, so that mostly laboratory data or prediction models are used instead. Reviews of water flux measurements and their limitations, including empirical and tracer methods, were presented by Tyler et al. (1999) and Flint et al. (2002). A major advantage of the MFHPP technique is that the water flux can be estimated indirectly from heat transport by convection, without the need for either soil water matric head or hydraulic conductivity measurements.

Temperature measurements have been used to measure water fluxes in other fields. For example, steady state measured temperature gradients in bore holes were used to estimate vertical water flow rates in groundwater, using the analytical solution of Bredehoeft and Papadopulos (1965), with minimum flow rates of about 30 cm $yr^{-1}$ using thermistors with a practical precision of about 0.01°C (Sorey, 1971). Using time-series measurements, thermal signatures in shallow estuarine sediments have been successfully used to estimate changes in nutrient-contaminated ground water recharge rates toward coastal waters (Land and Paull, 2001) and to estimate percolation rates in deep vadose zone environments (Constantz et al., 2003). Early application of the heat pulse technique to estimate water flow originates in the plant science literature, where advective transport of heat by water was used to indirectly estimate sap flow rates in trees (Marshall, 1958), mostly measuring flow magnitudes of 1 m $d^{-1}$ or larger.

The proposed MFHPP originates from the dual-probe heat-pulse (DPHP) method, introduced by Campbell et al. (1991). The DPHP method to estimate soil thermal properties and soil water content was experimentally tested by Bristow et al. (1993, 1994b), whereas measurement errors were analyzed by Kluitenberg et al. (1993, 1995). Successful application of the DPHP method has been demonstrated in both laboratory (Bristow et al., 1994b; Bilskie et al., 1998; Basinger et al., 2003) and field settings (Tarara and Ham, 1997; Ochsner et al., 2003; Heitman et al., 2003). Recent developments have led to the simultaneous measurement of soil thermal properties, water content, and electrical conductivity (EC) by using a thermo-TDR sensor, which combines time domain reflectometry (TDR) with the heat pulse probe (Ochsner et al., 2001; Ren et al., 2003). Alternatively, Bristow (1998) applied the HPP method to estimate simultaneously the soil's volumetric heat capacity, thermal conductivity, as well as volumetric water content. Bristow et al. (2001) presented the inclusion of the HPP with two additional needles for bulk soil EC measurements by a Wenner array, thereby allowing simultaneous measurement of soil solution concentration. Lastly, Cobos and Baker (2003) demonstrated how a

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**Abbreviations:** CV, coefficient of variation; DPHP, dual-probe heat-pulse; EC, electrical conductivity; HPP, heat pulse probe; MFHPP, multi-functional heat pulse probe; $R$, correlation coefficient; RMSE, root mean square error; TDR, time domain reflectometry.

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three-needle HPP could be used to accurately measure soil heat flux.

Ren et al. (2000) were among the first to report indirect estimates of soil water flux density, using the maximum temperature difference of the thermal responses between the upstream and downstream sensors of a three-needle HPP. Their application was conducted in the range of $1.2 \times 10^{-5}$ to $6.3 \times 10^{-5}$ m s$^{-1}$ (1.0–5.4 m d$^{-1}$) for steady state saturated flow conditions only. The complex mathematical relationship proposed by Ren et al. (2000) was later replaced with the much simpler approximation of Wang et al. (2002). Hopmans et al. (2002a) demonstrated an inverse technique by which thermal properties, water content, and water fluxes may be determined from temperature measurements, by including both conductive and convective heat transport in the two-dimensional heat flow equation, while accounting for dispersive heat transport caused by pore water flow variations. Most recently, Mori et al. (2003) demonstrated the simultaneous measurement of bulk soil water content, thermal properties, and EC using the so-called MFHPP for both saturated and unsaturated soils. The probe consisted of six sensors, including a heater, four thermistors, and a four-electrode Wenner array. This study also showed the potential application of the MFHPP to estimate soil water flux. Instead of using the Ren et al. (2000) solution, Mori et al. (2003) estimated water flux density from the fitting of the simple approximation of Wang et al. (2002). However, preliminary results showed a significant underestimation of soil water flux for fluxes smaller than 0.7 m d$^{-1}$, which was partly attributed to Wang's solution that did not account for differences in effective needle spacing between the upstream and downstream thermistor sensors. Moreover, this study showed high uncertainties of water flux values in unsaturated soils.

This study extends the work of Mori et al. (2003). Rather than using a single point temperature differences, we present a parameter optimization approach that uses a complete time series of both upstream and downstream thermistors to improve the accuracy of unsaturated water flux estimations, while extending the range to flux density values to smaller than 0.1 m d$^{-1}$. We will also demonstrate that thermal properties and volumetric water content can be estimated simultaneously from the continuous temperature response data of the horizontal thermistors, transverse to the water flow direction. The principal objective of this study was to determine the minimum water flux and maximum possible range of accurate soil water flux measurements using MFHPP measurements under both saturated and unsaturated soil conditions.

## MATERIALS AND METHODS

The HPP was originally developed for estimation of soil thermal properties through measurement of the rate of dissipation of an induced heat pulse by conduction (Campbell et al., 1991; Bristow et al., 1994a). To extend the HPP application for water flux measurements, convective heat transport must be considered. For uniform vertical water flow conditions, the simplified two-dimensional soil heat flow equation for a homogenous porous medium equation is written as (Bear, 1972; Hopmans et al., 2002a):

$$\frac{\partial T}{\partial t} = \kappa\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right] - V_h \frac{\partial T}{\partial z} \qquad [1]$$

where $T$ (K) is soil temperature as a function of time, $t$(s), and spatial positions $x$ and $z$ (cm), and $\kappa$ denotes the effective thermal diffusivity (m$^2$ s$^{-1}$), defined by the ratio of thermal conductivity, $\lambda$ (W m$^{-1}$ K$^{-1}$), and the soil's volumetric heat capacity, $C$ (J m$^{-3}$ K$^{-1}$).

The heat pulse velocity, $V_h$ (cm s$^{-1}$), describes convective heat flow by the moving water phase, relative to the stationary bulk porous medium (Ren et al., 2000), and is related to the Darcy water flux density, $J_w$ (cm s$^{-1}$), by

$$J_w = \frac{C}{C_w} V_h \qquad [2]$$

assuming that the soil's solid and fluid phases are in thermal equilibrium. The bulk soil volumetric heat capacity ($C$) is a function of the heat capacity of water ($C_w$) and the soil minerals, organic material, soil porosity and volumetric water content, $\theta$ (m$^3$ m$^{-3}$).

### Thermal Properties ($C$ and $\kappa$) and Volumetric Water Content ($\theta$)

Thermal property estimation using HPP method is based on a solution of the heat conduction equation (Eq. [1] with $V_h = 0$) for an infinite line heat source in a homogeneous and isotropic medium that is initially at uniform temperature. For a heat pulse of duration $t_0$(s), the solution for the temperature change, $\Delta T$(K), relative to the background temperature, at a distance $r$ (m) from the line heat source is given by (de Vries, 1952; Kluitenberg et al., 1993; Bristow et al., 1994a):

$$\Delta T(r,t) = \frac{q'}{4\pi C\kappa}\left\{\mathrm{Ei}\left[\frac{-r^2}{4\kappa(t - t_0)}\right] - \mathrm{Ei}\left(\frac{-r^2}{4\kappa t}\right)\right\};\ t > t_0 \qquad [3]$$

where $q'$ is the energy input per unit length of heater per unit time (W m$^{-1}$), and $-$Ei is the exponential integral function with argument $x$ (Abramowitz and Stegun, 1972). The parameter $r$ defines the spacing between heater and temperature sensors. Whereas $q'$ and $t$ can be measured accurately, the direct, in situ measurement of $r$ is difficult, realizing that solution of Eq. [3] is highly sensitive to variations in sensor spacing (Kluitenberg et al., 1993). Mori et al. (2003) recommended determination of effective sensor spacing, $r_{eff}$, by fitting Eq. [3] to temperature measurements of the saturated soil.

Given a measurement of $C$, the value of $\theta$ can be determined from the expression (de Vries, 1963; Campbell, 1985):

$$C = \rho_b c_s + C_w \theta \qquad [4]$$

assuming that the specific heat of air is negligible and the specific heats of the solid phase and water are available. In Eq. [4], $\rho$ denotes the material density (kg m$^{-3}$), $c$ is the specific heat (J kg$^{-1}$ K$^{-1}$), $C_w = \rho_w c_w$ and $b$, $s$, and $w$ denote bulk soil, solid phase and water, respectively.

Equation [3] was fitted to the temperature response of each of the two horizontal thermistors (1 and 3 in Fig. 1). Using nonlinear optimization, the residuals between measured and predicted $\Delta T(t)$ curves for the first 2 min after heating were minimized from

$$\mathrm{OF_I} = \sum_{i=1}^{N_h} [\Delta T_m(t_i) - \Delta T_o(t_i, \mathbf{p_I})]^2 \qquad [5]$$

where $\mathrm{OF_I}$ is the objective function, and subscripts m and o

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refer to the measured and optimized temperatures for the two horizontal thermistors, respectively. The constant $N_h$ denotes the number of measurement points, and the parameter vector $\mathbf{p_I}$ contains the optimized parameters. Average parameter values were computed from the fitting of the temperature data for each of the two thermistors. Equation [3] was fitted by minimizing Eq. [5] to determine the value of $r_{eff}$ for each thermistor sensor (calibration), assuming known values of $q'$ and $C$. After calibration, the known $r_{eff}$ value was substituted in Eq. [3], to estimate soil thermal properties and volumetric water content. In the calibration phase, $r_{eff}$ was determined for static saturated conditions. The validity of using Eq. [3] was tested to estimate thermal properties for a wide range of water flux values.

## Water Flux Density ($J_w$)

Solution of Eq. [3] is only valid for conductive heat transport. Ren et al. (2000) presented an analytical solution for Eq. [1] that includes convective heat transport, allowing estimation of the heat pulse velocity, $V_h$, or

$$\Delta T_u = \frac{q'}{4\pi C\kappa}\left\{\int_{t-t_0}^{t} s^{-1}\exp\left[-\frac{(r_u + V_h s)^2}{4\kappa\ s}\right]ds\right\};\ t > t_0 \quad [6a]$$

$$\Delta T_d = \frac{q'}{4\pi C\kappa}\left\{\int_{t-t_0}^{t} s^{-1}\exp\left[-\frac{(r_d - V_h s)^2}{4\kappa\ s}\right]ds\right\};\ t > t_0 \quad [6b]$$

where $s$ is the variable of integration, and $r_u$ and $r_d$ are the effective distance of the upstream ($T_u$) and downstream ($T_d$) thermistor from the heater needle, respectively. With a change of variables, it can be shown that the integrals in Eq. [6] are equivalent to the well function for leaky aquifers (Kluitenberg and Warrick, 2001).

Rather than using a single maximum temperature difference as presented by Ren et al. (2000), we present a parameter optimization approach that uses the time series of both upstream and downstream thermistors for 70 s. The resulting objective function for flux estimation was

$$\mathrm{OF_{II}} = \sum_{i=1}^{N_d}[\Delta T_{d,m}(t_i) - \Delta T_{d,o}(t_i,\mathbf{p_{II}})]^2 + \sum_{i=1}^{N^U}[\Delta T_{u,m}(t_i) - \Delta T_{u,o}(t_i,\mathbf{p_{II}})]^2 \quad [7]$$

where the parameter vector $\mathbf{p_{II}}$ now contains the unknown heat flux velocity ($V_h$) from which the water flux density can be computed using Eq. [2]. However, we will also show that $\kappa$ can be estimated simultaneously with the water flux density, if $C$ and $\theta$ are known a priori. Values of $V_h$ were optimized by minimizing Eq. [7]. The required evaluation of Eq. [6] was accomplished by using the method of Kluitenberg and Warrick (2001) to numerically evaluate the well function for leaky aquifers. Values for RMSE, CV, and correlation coefficients were computed for the $\log_{10}$ flux values, to account for the three orders of magnitude range of the measured flux density values.

## Multi-Step Outflow Method ($\theta$, $h$, $K$)

The measurements of soil thermal properties, water content, and water flux were combined with a single multi-step outflow experiment (Eching et al., 1994; Hopmans et al., 2002b) to estimate the soil water retention, $\theta(h)$, and unsaturated hydraulic conductivity, $K(\theta)$, functions. Soil water retention data were fitted with the van Genuchten (1980) model,

$$S_e = (1 + |\alpha h|^n)^{-m} \quad [8a]$$

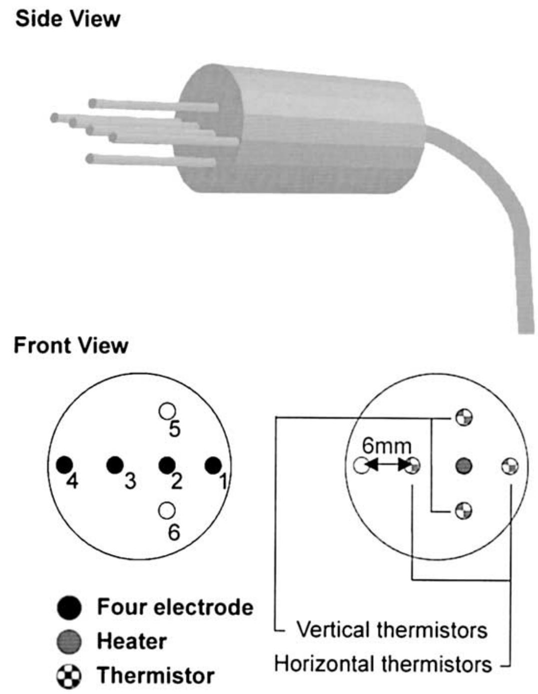


**Fig. 1. Schematic of multi functional heat pulse probe.**

$$S_e = (\theta - \theta_r)/(\theta_s - \theta_r) \quad [8b]$$

whereas the unsaturated hydraulic conductivity was described by the pore-size distribution model of Mualem (1976) to yield (van Genuchten, 1980):

$$K(\theta) = K_s S_e^l[1 - (1 - S_e^{1/m})^m]^2, \quad [9]$$

In Eq. [8] and [9], $S_e$ denotes the effective saturation ($0 \leq S_e \leq 1$); $h$ is the soil water matric head (cm); $\theta_r$ ($m^3\ m^{-3}$) is the residual water content; $\theta_s$($m^3\ m^{-3}$) is the saturated water content; $K_s$ ($m\ s^{-1}$) is the fitted saturated hydraulic conductivity; and $\alpha$ ($cm^{-1}$), $n$, $m$ ($m = 1 - 1/n$), and $l$ (assumed to be 0.5) are empirical parameters. The objective function for multi-step outflow experiment is

$$\mathrm{OF_{III}} = \sum_{i=1}^{N}\{w_i[Q_m(t_i) - Q_o(t_i,\mathbf{p_{III}})]\}^2 + \sum_{j=1}^{M}\{w_j v_j[h_m(t_j) - h_o(t_j,\mathbf{p_{III}})]\}^2 \quad [10]$$

where $Q$ is cumulative outflow, $t$ is time, and $w$ and $v$ are weighing parameters, assigning approximate equal weight between the two measurement types of size $N$ and $M$ and allowing for differential weighting of each data point. However, no weighting was used in the presented experiment, as outflow volume and matric pressure head values were of similar magnitudes. The soil water matric head was measured in the center of the soil core from tensiometric measurements during the outflow process. The parameter vector $\mathbf{p_{III}}$ included the optimized soil hydraulic parameters. The SFOPT code (Tuli et al., 2001) was used for the optimization. Soil water matric head, volumetric water content and water flux density values were computed over the length of the soil core, using a forward solution of the water flow equation with the optimized soil hydraulic parameters, so that these could be compared with MFHPP water flux density measurements.

## Multi-Functional Heat Pulse Probe (MFHPP)

The MFHPP (Fig. 1) consists of six parallel sensors with a spacing of approximately 6 mm between them. Sensor 2 serves as the heater. Temperature responses were measured by four thermistors (horizontal Sensors 1 and 3; vertical Sensors 5 and 6), located in the center of each needle, at approximately equal radial distances of about 6 mm from the heater sensor. The heat input, $q'$, was determined from the measurement of the voltage drop across a current sensing resistor in the heater circuit during the 8-s heating period. Sensors 1, 2, 3, and 4 combined make up a four-electrode Wenner array that can be used for bulk soil electrical conductivity measurements. The whole unit has a diameter of about 2.7 cm. See Mori et al. (2003) for details regarding design, installation and operation of the MFHPP.

### Outflow Experiment

The MFHPP was inserted through the wall of a 10-cm long and 7.9-cm i. d. Plexiglas column, with Thermistors 1 and 3 placed horizontally and Thermistors 5 and 6 oriented vertically. The outflow experiment was conducted for a Tottori Dune sand (Inoue et al., 2000), as it allowed for a wide range of water content and associated large range of water fluxes. The sand was washed to minimize clogging of the porous membrane by organic matter and/or clay-sized particles. The particle density was 2.67 Mg m$^{-3}$. The specific heat value of 795 J kg$^{-1}$ K$^{-1}$ was measured by differential scanning calorimetry, DSC (Kay and Goit, 1975) at 20°C (Mori et al., 2003), from replicate 30-mg samples.

A miniature tensiometer (Eching and Hopmans, 1993), to measure $h$, was inserted at the same height, but directly opposite the MFHPP. The sand was wet-packed in the column (Robinson and Friedman, 2001) with a known volume of water to achieve a saturated water content $\theta_s$ of 0.371 cm$^3$ cm$^{-3}$ and a uniform bulk density of 1.63 g cm$^{-3}$. This wet-packing approach was needed to achieve complete initial saturation and to avoid soil contact problems of the MFHPP sensors. A multi-step outflow experiment was conducted using standard procedures (Tuli et al., 2001), using pressure steps of 10, 20, 25, 30, 35, 40, and 50 cm. After drainage at the 50-cm pressure step was complete, the soil was oven-dried and the column-average water content for each measurement time was calculated by simple mass balance using the measured drainage data. The MFHPP measurements were performed every 5 min for the first 30 min of drainage as well as at hydraulic equilibrium for each pressure increment.

In a separate experiment, after repacking the sand with the same bulk density, a sequence of 35 saturated water flux experiments was conducted. A wide range of water flow densities between 0.056 and 27 m d$^{-1}$ was achieved by changing the hydraulic head of water in a 60-cm long Plexiglas tubing that was fitted on top of the sand-filled core (Fig. 2), while varying the height of the drip needle at the drainage end. The water flux was determined assuming steady state during 2-min periods, while measuring drainage volumes from weight measurements. Estimated errors as caused by the steady state assumption ranged between 0.25 and 2.3% for fluxes of 0.056 and 27 m d$^{-1}$, respectively. We note that different heat pulse probes were used for the outflow (MFHPP2) and saturated flow experiments (MFHPP4), so that experiments could be done simultaneously. Effective $r$-values for all four thermistors of both probes were obtained at static saturated conditions by in situ calibration.

## RESULTS AND DISCUSSION

### Calibration

After saturation of the soil core, the effective sensor spacing, $r_{\mathrm{eff}}$, for all four thermistors was determined by minimizing the objective function Eq. [5], from the fitting of Eq. [3] to the temperature response data of each

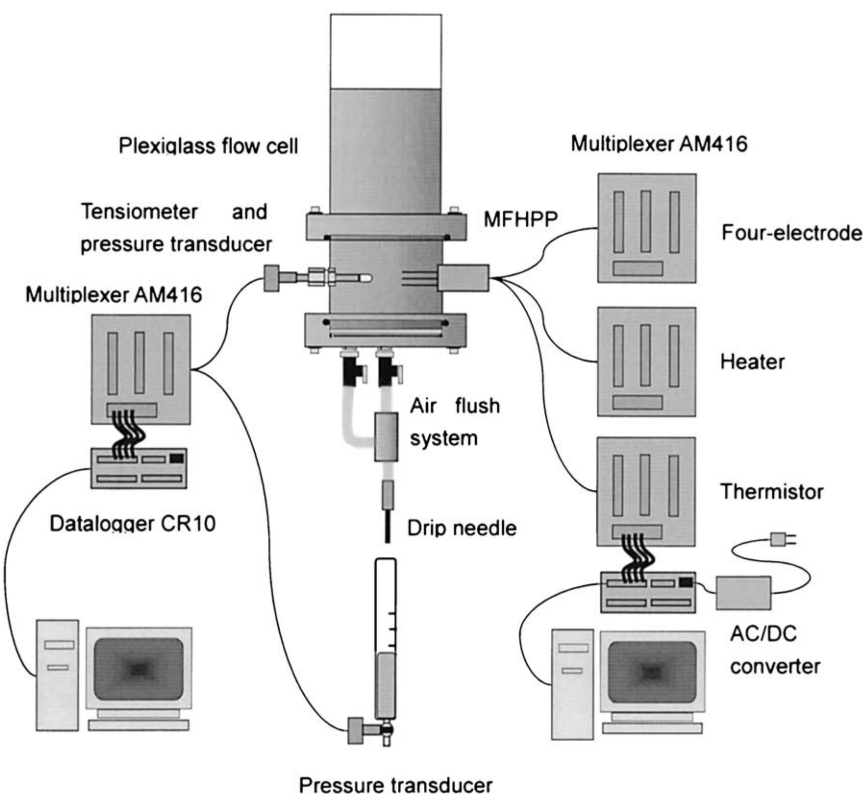


**Fig. 2. Experimental setup of saturated flow experiment.**

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thermistor separately. This in situ calibration was done as described in Mori et al. (2003), using known values of the saturated water content, bulk density, and the specific heat of water and sand. It also required κ as an additional fitting parameter. Resulting values for $r_{eff}$ were 5.833, 5.983, 5.785, and 6.057 mm for thermistors 1, 3, 5, and 6 (Fig. 1) of MFHPP2 and 6.010, 6.143, 5.753, and 5.5505 mm for the respective thermistors of MFHPP4. Corresponding average fitted κ values for the saturated sand were $6.5 \times 10^{-7}$ and $6.7 \times 10^{-7}$ $m^2$ $s^{-1}$, respectively.

## Thermal Diffusivity

Fitting the temperature responses of the horizontal thermistors to Eq. [3] during the outflow experiment yielded thermal diffusivity as a function of volumetric water content, during both transient (solid circles) and no-flow (open circles) conditions. These data are compared with those reported by Mori et al. (2003) in Fig. 3. The water content values at the equilibrium points were calculated from the cumulative drainage measurements, assuming that the core-average volumetric water content is equal to the water content in the center of the core. When fitting Eq. [3] to the temperature responses of the horizontal thermistors for the high saturated-flow experiments (gray circles), the thermal diffusivity was largely overestimated. This is caused by attenuation of the thermal signal by the convecting water. The effect of water flux on the measured thermal diffusivity for saturated conditions is presented in Fig. 4, with the solid horizontal line representing the average value as measured from the saturated soil core with no water flow (static case), using a fitted κ value equal to $6.5 \times 10^{-7}$ $m^2$ $s^{-1}$. In both graphs we purposely excluded the much larger κ overestimations at water flux values larger than 10 m $d^{-1}$ that approach one order of magnitude error at a water flux density of 27.0 m $d^{-1}$. Specifically, Fig. 4 demonstrates that the temperature signals of the horizontal thermistors can be used if the water flux is less than about 2 m $d^{-1}$. The effects of neglecting convective heat transport on $C$ and θ estimations by applying the solution of Eq. [3] rather than Eq. [6] were presented by Kluitenberg and Heitman (2002). The case of using the horizontal thermistors in our study corresponds with orientation I of theirs. Also Kluitenberg and Heitman (2002) showed that the effect of convective heat transport on the temperature response for orientation I was relatively small for water flux values less than about 3.0 m $d^{-1}$ for their Hanlon sand.

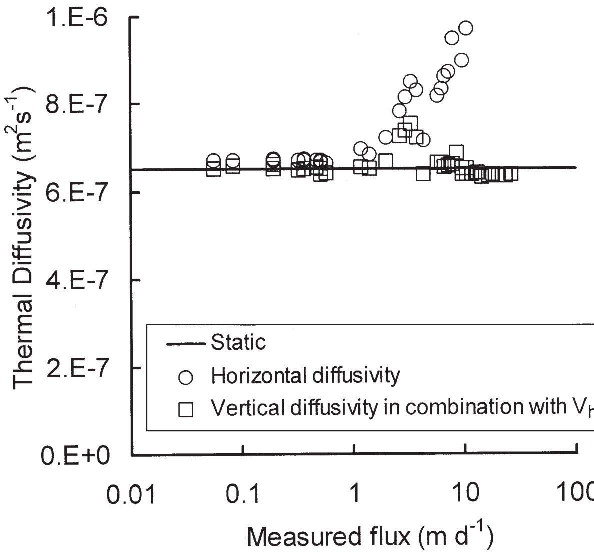


**Fig. 4. Optimized thermal diffusivity (κ) values as a function of water flux density: (circles) using Eq. [3] and (squares) using Eq. [6]. Data for water flux values larger 10 m $d^{-1}$ are omitted.**

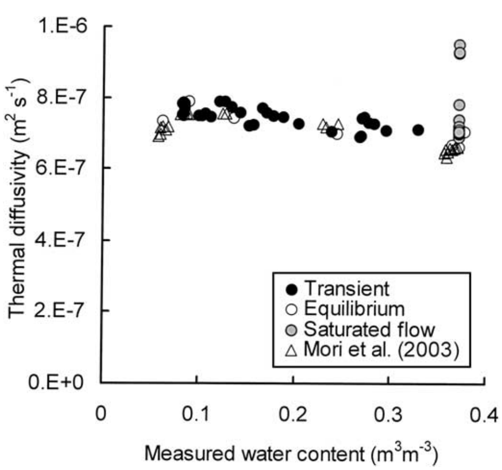


**Fig. 3. Thermal diffusivity values as a function of volumetric water content. Data for water flux values larger 10 m $d^{-1}$ are omitted.**

## Volumetric Water Content

The comparison between estimated and measured water content values is shown in Fig. 5, making distinction between data points obtained from the hydraulic equilibrium points (between pressure increments) and during transient flow with fluxes ranging between 0.00058 to 1.4 m $d^{-1}$. The estimated water content data are average values, obtained by fitting the temperature response of each horizontal thermistor separately to Eq. [3], with θ computed from Eq. [4]. The MFHPP measurements are compared with simulated water content values using

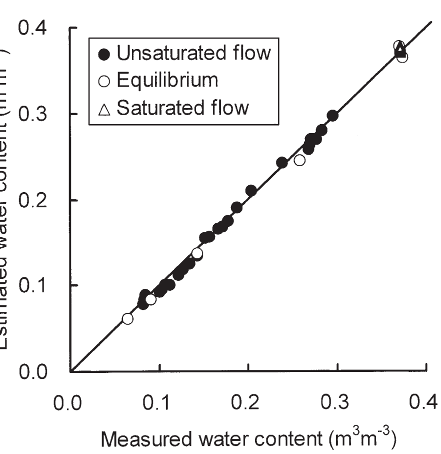


**Fig. 5. Comparison of estimated (with MFHPP) and independently measured water content values. Only saturated water content values for a water flux <0.5 m $d^{-1}$ are included.**

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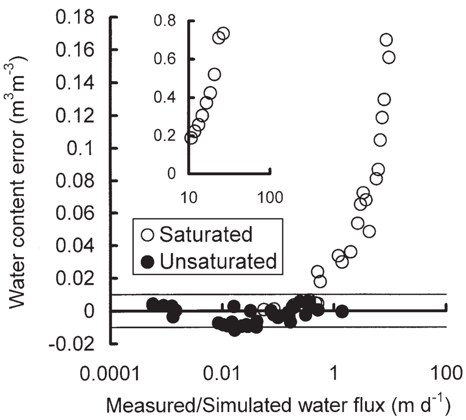


**Fig. 6. Accuracy of water content estimation by Eq. [3], as a function of water flux density.**

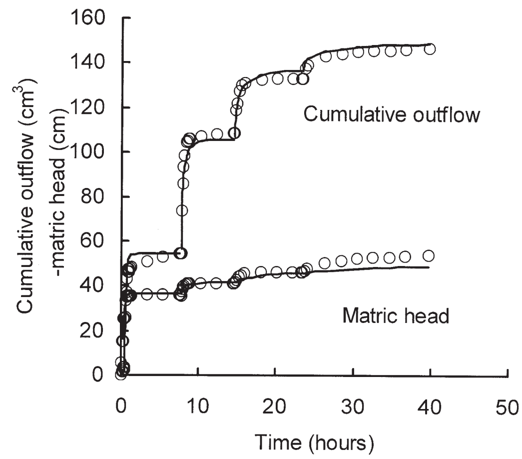


**Fig. 7. Comparison of measured with optimized matric head (cm) and cumulative outflow ($cm^3$) for multi-step outflow experiment.**

SFOPT for the transient data. As a side note, the simulated water content data corresponded very well with the water content data as inferred from the measured drainage volumes. Correlation coefficient and RMSE values for all data in Fig. 5 were 0.998 and 0.0056 $m^3$ $m^{-3}$ ($n$ = 32), respectively. For reasons unclear, these results were even better than the MFHPP data in Mori et al. (2003), who reported a RMSE value of 0.014 $m^3$ $m^{-3}$. Also, in general, the MFHPP results are better than reported for laboratory experiments using the DPHP by Basinger et al. (2003) and using the Thermo-TDR by Ren et al. (2003). However, our results are limited to those for a single Tottori sandy soil only. For reasons that are unclear to date, our results do not show a bias for the lower water content range as reported by Basinger et al. (2003). For comparison, the same Fig. 5 also includes the estimated saturated water content values for the saturated flow experiments (triangles) with flow rates less than 0.5 m $d^{-1}$. To highlight the effect of convective water flow on the temperature signal of the horizontal thermistors, we show the volumetric water content error as a function of water flow velocity in Fig. 6. The insert identifies increasing measurement errors for water flux values approaching 30 m $d^{-1}$. Our measurements show that water content errors are larger than 1 volume percent for water flux values greater than 0.5 m $d^{-1}$. This water flux sensitivity is larger than reported by the theoretical study for the Hanlon soil of Kluitenberg and Heitman (2002). They concluded that the theoretical error in the water content measurement using temperature responses for placement of the thermistors in a plane perpendicular to the direction of water flow (transverse direction), is smaller than 1 volume percent for convective water fluxes smaller than 3 m $d^{-1}$. However, the reported theoretical and measured errors are close at the higher water fluxes, both approaching 8 to 10 volume percent at water flux values near 10 m $d^{-1}$. These overestimations are caused by the reduction in maximum temperature rise as a result of the vertical water flow component, convecting heat downward and away from both horizontal thermistors.

## Water Flux Density

For the unsaturated outflow experiment, the optimized soil hydraulic functions were used to estimate the water flux in the center of the soil core. Figure 7 compares the measured with the fitted outflow and head data, using the following optimized parameter values: $\alpha = 0.0272$ $cm^{-1}$, $\theta_r = 0.0581$ $cm^3$ $cm^{-3}$, $n = 13.02$, and $K_s = 15.80$ cm $h^{-1}$ (3.8 m $d^{-1}$). The fitted $K_s$ value was about five times smaller than the independently measured value of 20.3 m $d^{-1}$, however, this result is commonly found when using unsaturated flow data to predict the saturated hydraulic conductivity (Hopmans et al., 2002b). Unsaturated hydraulic conductivity is controlled by the soil matrix, whereas the saturated conductivity is mostly controlled by macropores. As the multistep outflow range is solely unsaturated, the extrapolation to saturation using Mualem's model underpredicts measured saturated hydraulic conductivity. The corresponding correlation coefficients between the optimized and observed data were 0.986 for matric head and 0.998 for cumulative outflow. Because of this excellent fit, we used the simulated fluxes in the center of the soil core as a reference, by which the estimated water flux values from the MFHPP temperature data could be compared.

To test the MFHPP as a sensor for accurate water flux measurements, we compared independently measured water fluxes with MFHPP-estimated water fluxes for saturated conditions using a saturated water content value of 0.371 $m^3$ $m^{-3}$ to compute the volumetric heat capacity in Eq. [4]. Only the thermal flux term ($V_h$) was optimized in Eq. [6], assuming that the thermal diffusivity ($\kappa$) value was equal to $6.5 \times 10^{-7}$ $m^2$ $s^{-1}$, as estimated from fitting Eq. [3] to the measured temperature response for the no-flow saturated core. The saturated flux estimation results are shown in Fig. 8 by the open symbols. The MFHPP-estimated water fluxes agreed very well with the directly measured fluxes, with correlation coefficient and RMSE values of 0.995 and 0.0952 log(m $d^{-1}$), respectively, across a wide range of fluxes between 0.056 to 27.0 m $d^{-1}$.

The results for the unsaturated flux estimations are shown in Fig. 8 as solid symbols. As for the unsaturated flow experiments, the estimated water flux density values for the unsaturated multi-step outflow experiments were obtained from fitting Eq. [6], using volumetric heat capacity and thermal diffusivity values from fitting the

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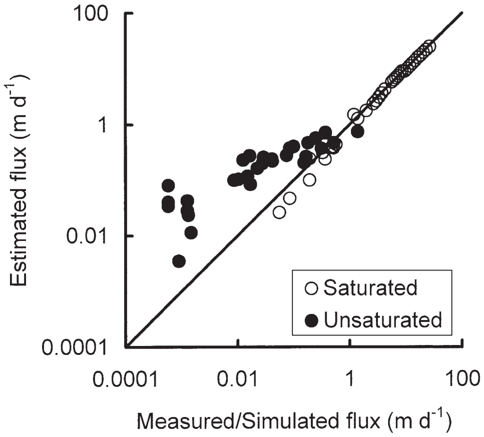


**Fig. 8. Comparison of estimated (with MFHPP) with measured (saturated) or simulated (multi-step outflow) water flux densities, using Eq. [6] for both saturated and unsaturated flow.**

temperature responses of the horizontal thermistors to Eq. [3].

The agreement of the MFHPP flux measurements with simulated fluxes during unsaturated conditions was close at near-saturation at the higher fluxes, but deviations increased as water fluxes decreased to $<0.1$ m d$^{-1}$, with corresponding water content values smaller than 0.24 m$^3$ m$^{-3}$. Nevertheless, values for the correlation coefficient and RMSE for unsaturated conditions were 0.873 and 0.974 log(m d$^{-1}$), respectively. When comparing these results with Fig. 11 of Mori et al. (2003), we conclude that the presented approach using continuous temperature data to fit Eq. [6a] and [6b] separately for the downstream and upstream thermistors, improved the minimum measurable water flow velocity from 1.0 to about 0.1 m d$^{-1}$ or smaller, irrespective of water saturation. This is an improvement of at least one order of magnitude from earlier studies. There are two possible reasons for the decrease in flux estimation accuracy with decreasing water flux density. First, irrespective of saturation, the maximum temperature resolution of the thermistors is limited. For example, Ren et al. (2000) estimated that the minimum water flux density that can be accurately determined is 0.09 m d$^{-1}$, if the temperature resolution is 0.01 K, or about 0.01 m d$^{-1}$ if the temperature resolution is 0.001 K. The accuracy of the employed thermistors is 0.01 K, with a precision of about 0.004 K (Mori et al., 2003). Therefore, we would not expect accurate flux estimates at water flow velocities smaller than about 0.1 m d$^{-1}$, unless we used thermistors with a higher temperature resolution. The smallest saturated water flux of 0.056 m d$^{-1}$ was close to the measurement limit of our temperature sensor. Second, whereas thermal heat transport is through the solid and liquid phase for saturated flow, conductive flow paths are expected to be much more complex for an unsaturated soil that include air, in addition to water and solid phases. As a result, the assumption that all phases are in thermal equilibrium is much closer to reality for the saturated than unsaturated case. At the same time, though, one would expect that water fluxes in unsaturated conditions are relatively low, favoring thermal equilibrium.

The presented approach for estimating water flux assumes that thermal diffusivity is known a priori. Although its value can be accurately obtained from the horizontal sensors at small water flux values, measurement errors in the thermal properties become unacceptable at water flow velocities larger than 2 m d$^{-1}$ for the Tottori sand used here (Fig. 4). Instead, for those conditions we estimated both $V_h$ and $\kappa$ from the fitting of Eq. [6], assuming known water content and volumetric heat capacity values from the static measurements. Since these high water flux values are likely to occur for saturated soil conditions only, we suggest that the saturated water content can be estimated from soil bulk density or is known a priori. We did not add the additional water flux estimations in Fig. 8, since results were identical with the saturated data already included, with correlation coefficient and RMSE values of 0.9949 and 0.09522 log(m d$^{-1}$), respectively.

The newly acquired thermal diffusivity values are included in Fig. 4 (open squares), and are compared with the relatively large errors from the fitting of the temperature responses to Eq. [3] (open circles). We note that the fitted thermal diffusivity is nearly independent of water flow and close to the estimated value that was obtained for the static saturated condition (solid horizontal line). Similarly, the results in Fig. 4 also indicate that thermal dispersion effects, as reported in Hopmans et al. (2002a), are insignificant for the water flux range of our experiments. Possibly, a sensor spacing of 6 mm is too small for thermal dispersion to fully develop.

## CONCLUSIONS

Water flux density was measured from MFHPP measurements of both static and transient flow in both unsaturated and saturated soil conditions. Using analytical solutions to the radial heat transport problem, water content and thermal diffusivity values were estimated from horizontally oriented thermistors, whereas water flux estimations were determined using continuous temperature measurements of the vertical-oriented thermistors of the MFHPP. We conclude that the temperature signals of the horizontal thermistors can be used to estimate soil thermal diffusivity for water flux density values less than about 2 m d$^{-1}$, whereas water content errors are smaller than 1 volume percent for water flux values $<0.5$ m d$^{-1}$. Flux estimations were excellent for saturated water flow for a wide range between 0.056 to 27 m d$^{-1}$. However, for unsaturated conditions, MFHPP estimations increasingly overestimated water flux density, as values decrease to smaller than 0.10 m d$^{-1}$. If the soil's volumetric water content is known a priori, such as for saturated soils, thermal properties and water flux can be estimated simultaneously for water flux values larger than 2 m d$^{-1}$. When comparing these results with those of Mori et al. (2003), we conclude that the presented approach improved the minimum measurable water flow velocity from 1.0 to about 0.1 m d$^{-1}$ or smaller, irrespective of water saturation.

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