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Optimization of Hydraulic Functions from Transient Outflow and Soil Water Pressure Data

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ABSTRACT

Inverse solution techniques currently used for estimating unsaturated soil hydraulic functions from laboratory outflow experiments use cumulative outflow only in combination witb initial and final soil water pressure head values. Additional soil water information is needed to improve the estimation procedure and to minimize uniqueness problems. It was the objective of this study to experimentally explore the feasibility of using both cumulative outflow and soil water pressure head data in the inverse solution for laboratory determination of soil hydraulic functions using both one-step and multistep outflow experiments. Soil water pressure head was measured with a microtensiometer and pressure transducer. Desorption experiments were performed under both pressure and suction for the following soils: Yolo silt loam (fine-silty, mixed, nonacid, thermic Typic Xerorthent), Panoche loam (fine-loamy, mixed [calcareous], thermic TypicTorriorthent), Hanford sandy loam (coarse-loamy, mixed, nonacid, thermic Typic Xerorthent), and Oso–Flaco fine sand (coarse-loamy, mixed Typic Cryorthod–fine-loamy, mixed, mesic Ustollic Haplargid). Water retention curves optimized from cumulative outflow alone were compared with those obtained from outflow and soil water pressure head measurements for both one-step and multistep outflow experiments. Computer optimization of the retention curve by the inverse solution technique using transient outflow experiments was greatly improved when cumulative transient outflow data were combined with simultaneously measured soil water pressure head data. Pressure and suction experiments yielded equally good results for one-step and multistep desorption. Moreover, the addition of soil water pressure head data resulted in unique parameter values for the optimized soil hydraulic functions under our experimental conditions.

PREDICTION OF WATER MOVEMENT in soil as well as the fate of salts and contaminants under different combinations of soil, climatic, and management conditions is dependent on availability of representative soil hydraulic properties. Both the hydraulic conductivity, K, as a function of water content, θ, or soil water pressure head, h, and the soil water retention function $\theta(h)$ influence the rate at which water and solute move in the vadose zone and thus affect travel time of contaminants toward groundwater.

Direct methods for the determination of these highly nonlinear functions exist (Klute, 1986; Klute and Dirksen, 1986). These methods, however, require the experiments to reach several stages of steady-state or equilibrium conditions. Moreover, they require restrictive initial and boundary conditions, which make them time consuming and expensive (van Dam et al., 1990). The parameter estimation technique involves the indirect estimation of soil hydraulic functions by numerical solution of the equation governing the flow process, subjected to the imposed boundary conditions. First, the hydraulic properties are assumed to be described by an analytical model with unknown parameter values. An experiment is set up under controlled conditions with prescribed initial and boundary conditions. During the experiment, one or more flow-controlled attributes (auxiliary variables) are measured. Subsequently, the flow equation is solved numerically using the parameterized hydraulic functions with initial estimates provided. The parameters of the hydraulic functions are optimized by minimization of the objective function containing the sums of squared deviations between observed and predicted auxiliary variables, using repeated numerical simulation of the flow process. This iterative inversion of the flow equation is in contrast to direct inversion techniques as used in analytical solutions.

By far, laboratory outflow experiments have been the most attractive for estimation of soil hydraulic functions by computer optimization. The outflow method was introduced by Gardner (1956) for determining the unsaturated soil water diffusivity and unsaturated hydraulic conductivity by analytical techniques. This analytical technique has since been

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modified (Gardner, 1962; Gupta et al., 1974), and more recently by Valiantzas (1989) and Valiantzas and Kerkides (1990). The use of outflow experiments for estimation of soil hydraulic functions is advantageous because it is flexible in initial and boundary conditions. Moreover, the technique yields fast results, and is relatively cheap. The methodology, however, has not been accepted yet. Among other things, problems arise because of nonuniqueness of the solution (Russo et al., 1991; Toorman et al., 1992), and the general need of fast-processing computers. Although the concept was simple, initially investigators were faced with the dilemma of deciding which auxiliary variable and parametric models to use. This led to the initial application of the inverse solution technique to numerical experiments, as by Zachman et al. (1981). In their numerical experiments, an initially saturated vertical soil column was allowed to drain. Variables considered were cumulative outflow as a function of time, soil water pressure head as a function of time at one location, and water content as a function of time also measured at a single location. Soil hydraulic functions by parameter estimation using cumulative outflow as a function of time were closest to the true hydraulic data.

The first application of the inverse problem to field data was carried out by Dane and Hruska (1983), in which parameters of van Genuchten's (1980) soil hydraulic functions were optimized from drainage data. Already then, Dane and Hruska (1983) questioned the uniqueness of their solutions. They also concluded that the sensitivity of optimized parameters depended on the employed boundary conditions. Parker et al. (1985) were the first to apply the inverse solution technique to the one-step outflow method. Kool et al. (1985a) found that, to minimize uniqueness problems, one would need to design an experiment covering a wide range in soil water content. Moreover, they also determined that initial parameters must be close to their true values, and that experimental error in outflow data must be small. Russo (1988) investigated the van Genuchten (1980), Brooks and Corey (1964), and Gardner–Russo (Russo, 1988) soil hydraulic models. He used the one-step optimization program (Kool et al., 1985b) and data from Kool et al. (1985a) and Parker et al. (1985) to estimate the parameters of the various models. Russo (1988) concluded that, of the various soil hydraulic models considered in the parameter estimation procedure of the one-step outflow method, the van Genuchten (1980) model was superior. Subsequently, Russo et al. (1991) also pointed out that the larger number of parameters in the van Genuchten (1980) model may enhance the likelihood of nonuniqueness and instability in the inverse solution.

Analysis of the objective function by Toorman et al. (1992) using van Genuchten's soil water retention model and hypothetical data sets indicated that uniqueness problems were minimized if soil water pressure data were included in the objective function of a transient one-step outflow experiment. Specifically, they concluded that parameter estimation sensitivity was improved if soil water pressure measurements were taken at some distance away from the lower boundary of the pressure cell. The advantage of including soil water pressure head data in the inverse method was also discussed by Kool and Parker (1988). In their hypothetical infiltration and redistribution experiment, pressure head measurements at a single depth were included in the objective function. The benefit of including soil water pressure data in combination with water content data is also intuitively clear, since the simultaneous measurement of these two soil attributes forces the optimized soil water retention curve to match the observed $\theta(h)$ data. Reasonable parameter estimates were also obtained by Hudson et al. (1991) in an upward-infiltration experiment in which soil water pressure head and water contents were used in the estimation procedure.

The difference between soil water retention data obtained by pressure and suction desorption was explored by Chahal and Yong (1965) and Peck (1960), and reviewed by Klute (1986). Discrepancies were attributed to changes in trapped air volumes while releasing the pressure, thereby causing the assumed soil water potential for equilibrated pressurized soil samples to be lower than for soil equilibrated under suction.

It was the objective of this study to extend the theoretical analysis of Toorman et al. (1992) to experimental data, specifically, to explore the feasibility of using both cumulative outflow and soil water pressure head data in the inverse solution for laboratory determination of soil hydraulic functions using both one-step and multistep outflow experiments. We examined the possibility of minimizing uniqueness problems and obtaining representative estimates of $\theta(h)$. For this purpose we experimented with the outflow method using four soils with a wide variety in texture. Moreover, we compared results from experiments with drainage induced by either a positive N pressure or by application of a suction at the outflow end of the soil sample. The central theme of this experimental study is similar to the work presented by Kool et al. (1987) and Kool and Parker (1988). These two studies demonstrated the usefulness and advantages of the parameter estimation technique by inverse solution of the water flow equation, using hypothetical and field data. Their work was critical in our study, where we focused on the application of the outflow method to soil cores in the laboratory, and present the methodology to make the inverse procedure under these conditions successful.

THEORY

The experimental procedure involves the measurement of cumulative outflow and soil water pressure head as a function of time from initially near-saturated soil cores with a saturated ceramic plate at the bottom. Soil samples were subjected to an instantaneous increase in pneumatic pressure at the top, or suction at the bottom below the ceramic plate. The equation describing the drainage as a result of the imposed soil water potential gradient in these cores, was assumed to follow Richards' equation. In its one-dimensional form with the vertical coordinate, z, taken positive downward, it is written as

$$c(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left[K(h)\left(\frac{\partial h}{\partial z} - 1\right)\right] \quad [1]$$

where $C = d\theta/dh$ is the water capacity and t is time. The combined system of soil and ceramic plate (two-layer system) has the following initial and boundary conditions:

$$h = h_o(z) \qquad t = 0 \qquad 0 \le z \le L \qquad [2a]$$

$$\partial h/\partial z = 1 \qquad t > 0 \qquad z = 0 \qquad [2b]$$

$$h = h_L - |h^a| \qquad t > 0 \qquad z = L \qquad [2c]$$

where $z = 0$ is the top of the soil core, $z = L$ is the bottom of the ceramic plate, h_L is the soil water pressure head at the bottom of the ceramic plate at $t = 0$, and h^a is either the pneumatic pressure applied to the top of the core or suction applied beneath the plate at $t > 0$. For example, if $h_L = 3$ cm and a pneumatic pressure head (h^a) of 80 cm is applied to the top of the core, then $h = -77$ cm, or, if a 80-cm suction head (h^a) is applied to the bottom of the ceramic plate in addition to h_L, again $h = -77$ cm. Thus, in its presented form, the boundary conditions are indifferent with respect to how pressure head gradients are created. The numerical solution of Eq. [1] and [2] was obtained by a modified version of the Galerkin finite element model of van Genuchten (1978).

The soil water retention curve was assumed to be of the form described by van Genuchten (1980):

$$S_e = (1 + |\alpha h|^n)^{-m} \qquad [3]$$

$$S_e = (\theta - \theta_r)/(\theta_s - \theta_r) \qquad [4]$$

with $m = 1 - 1/n$, and S_e is the effective saturation ($0 \le S_e \le 1$), θ_r (m^3 m^{-3}), and θ_s (m^3 m^{-3}) are the residual and saturated water contents, respectively, and α (cm^{-1}), n, and m are empirical parameters. An expression for the unsaturated hydraulic conductivity function is obtained by combining Eq. [3] with the pore-size distribution model of Mualem (1976) to yield (van Genuchten, 1978):

$$K(\theta) = K_s S_e^l [1 - (1 - S_e^{1/m})^m]^2 \qquad [5]$$

where l (assumed to be 0.5) is an empirical parameter and K_s (cm h^{-1}) is the saturated hydraulic conductivity. The use of Eq. [3] and [5] in the inverse solution of Eq. [1] implies that optimization of the parameters α, n, θ_r, and K_s will yield a numerical solution that matches experimental outflow and soil water pressure head observations. Thus, a priori, we selected the van Genuchten hydraulic relationships and assumed that these adequately describe soil water behavior in a draining soil core. In order to simulate cumulative outflow and soil water pressure head in the optimization of the parameters of the hydraulic models, we modified the program MULSTP (van Dam et al., 1990), which is based on ONESTEP (Kool et al., 1985b). The MULSTP program allows some or all of the unknowns, θ_r, θ_s, α, n, l, and K_s to be optimized simultaneously. It also allows for stepwise changes in the lower boundary condition during the transient experiment. Furthermore, uniqueness of an optimized solution is tested by two additional optimizations with initial parameter values automatically generated by the program. This approach of uniqueness testing a posteriori is heuristic, but necessary to ascertain uniqueness. The MULSTP program was adapted to include soil water pressure head data in addition to outflow measurements in the objective function as described by Kool and Parker (1987). As in ONESTEP, the least squares parameter optimization in MULSTP is based on Marquardt's maximum neighborhood method (Marquardt, 1963). The objective function O (**b**) to be minimized in the revised algorithm is

$$O(\mathbf{b}) = \sum_{i=1}^{N} \{W_i[Q_o(t_i) - Q_c(t_i,\mathbf{b})]\}^2 + \sum_{j=1}^{M} \{W_j V_j[h_o(t_j) - h_c(t_j,\mathbf{b})]\}^2 + \sum_{k=1}^{L} \{W_k V_k[\theta_o(h_k) - \theta_c(h_k,\mathbf{b})]\}^2 \qquad [6]$$

where **b** is a vector containing the optimized parameters θ_r, θ_s, α, n, l, and K_s. In our experiments we measured θ_s and assumed the value of l to be 0.5. Hence, **b** contained four parameters only. The variable Q denotes cumulative transient outflow volume (cm^3), h and θ are defined as before, and subscripts o and c denote observed and calculated values, respectively; N, M, and L represent the number of cumulative outflow, pressure head, and soil moisture content measurements during the outflow experiment, respectively. In our experiments, only h values measured simultaneously with outflow data were included in Eq. [6]; hence, N is equal to M . Of course, for the cases without soil water pressure measurements, M was equal to zero. As only the initial moisture content was used in the objective function of our experiments, L was equal to one. The variables W and V are weighting coefficients (Kool and Parker, 1987); W may be used to weigh each measured data point individually. It was set to $W = 1$ for the cumulative outflow and pressure head data, but a value of $W = 5$ was assigned to the initial soil moisture content value of the soil sample. This differential weighting ensured that the initial condition is part of the optimized retention curve. Variable V is calculated internally in the program so that the soil water pressure head and soil moisture data are weighted equally as the cumulative outflow data (Kool and Parker, 1987).

MATERIALS AND METHODS

Outflow experiments were carried out for the Yolo silt loam, Panoche loam, Hanford sandy loam, and Oso–Flaco fine sand (Luthin and Day, 1955). Soils were air dried, sieved through a 2-mm screen, and packed to a uniform predetermined bulk density in 8.2- cm-diam. and 6-cm-length brass cylinders assembled in modified pressure cells (J.H. Dane, 1985, personal communication) with 0.57-cm-thick, 1000-cm air-entry ceramic plates (Fig. 1). Results of the particle-size analyses and bulk density are presented in Table 1. Figure 1 shows the arrangement that enabled both pressure and suction desorption experiments to be conducted on the same soil core. Pressure was applied through P with Valve 1 open and Valve 2 closed. In the suction experiment, on the other hand, suction was applied through S with Valve 1 closed and Valve 2 open. Microtensiometers were constructed by cementing a 1-cm-long, 0.635-cm-o.d. high-flow ceramic cup (Soil Moisture Corp., Santa Barbara, CA) to a 9-cm-long, 0.63-cm-o.d. acrylic tube. The acrylic tubing was connected to the transducer (Beswitched, Phoenix, AZ) by a short piece of Tygon tubing to make a rigid connection. To prevent damage to the transducer membrane, the connection was done slowly, allowing water to escape through the ceramic cup. Each transducer was calibrated independently across the entire pressure or suction range before and after each experiment using linear regression. The transducers were connected to a 21× micrologger (Campbell Scientific, Logan UT) and an independent 8 V direct current excitation source through a modular-type telephone cable. An external excitation voltage was used to prolong the battery life of the micrologger. The cover of a modified pressure cell was modified to accommodate the vertical tensiometer (Fig. 1). Shown in Fig. 1 is an enlargement of the attachment that consists of a compression link with a pipe thread. The O-ring in the assembly provides a pressure-tight seal and yet allows the

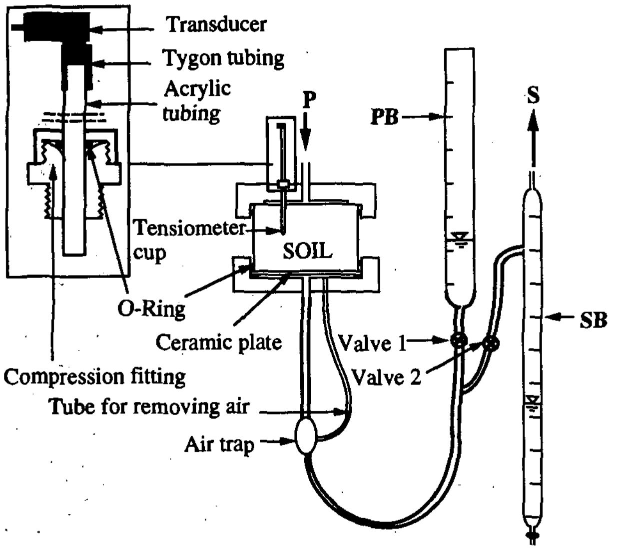

Fig. 1. Modified pressure cell (P, pressure inlet port; S, suction inlet port; PB, burette for pressure experiment; SB, burette for suction experiment).

tensiometer to be adjusted when the link is not fully tight. Before attaching the cover with the tensiometer to the cell, soil samples were saturated by wetting from the bottom to saturation. Deaerated 0.01 *M* $CaCl_2$ solution was used to minimize dispersion. After saturation, the cell cover was attached to the cell and samples were equilibrated with an initial soil water pressure head of −30 cm at the center of the sample with cumulative outflow measured. The motivation to start the experiment at soil water pressure below saturation was presented by Hopmans et al. (1992). They argued that Richards' equation, which is employed in the optimization technique, assumes air continuity everywhere in the sample at all times. The presence of a ceramic plate at the outflow end of a saturated soil sample prevents the existence of a continuous air phase during initial desorption. They suggested that better results are obtained with the outflow method if the sample is initially unsaturated. Outflow experiments were carried out under both suction and with pressurized samples using N_2 gas. All experiments were performed at 21 °C in a constant-temperature room. The following experiments were performed:

Experiment A. Step-wise equilibrium desorption experiments were performed for this experiment (Klute, 1986). Pressure and suction head increments for the Panoche and Hanford soils were 30, 80, 200, 400, and 700 cm. For the Yolo soil the increments were 30, 60, 100, 200, 400, and 700 cm. Increments of 30, 40, 80, and 200 cm were used for the Oso–Flaco fine sand. Equilibrium outflow volume and soil water pressure head were measured at each step. These retention data served as a reference by which the optimized retention functions were compared.

Experiment B. One-step transient outflow experiments with a pneumatic head or suction increment of 700 cm were performed (Kool et al., 1985b). The Oso–Flaco fine sand was subjected to a pressure or suction of 400 cm.

Experiment C. Multistep transient outflow experiments were performed (van Dam et al., 1990) with pressure or suction head steps of 40, 60, 80, 200, 400, and 700 cm. The Oso–Flaco fine sand was subjected to steps of 40, 60, 80, and 200 cm. The shape of the cumulative outflow curve was monitored during the experiment. Pressure or suction was incremented when the cumulative outflow curve approached a plateau (outflow rate of ≈0.05 mL h^{-1}), a period defined by Passioura (1976) as the third stage of the cumulative outflow curve.

In all transient experiments, cumulative outflow volume and soil water pressure head were measured as a function of time. Soil water pressure head in the pressure experiments was computed as the difference between the measured soil water pressure head and the applied pneumatic pressure, while outflow was measured with burette PB (Fig. 1). The burette was arranged such that the water level could be adjusted to provide a constant pressure below the ceramic plate. The two ports at the bottom of the cell and the air trap permitted the flushing of trapped air under the ceramic plate. The experimental apparatus and technique was similar to the one presented by Kool et al. (1985b). In the suction experiment, the inverted constant-head burette SB (Fig. 1) was used to measure outflow, while simultaneously connected to the suction source through port S. Pressure head readings were recorded automatically at 1-min intervals for the full duration of the experiment. However, only those readings taken at times corresponding to 3-mL cumulative outflow volume increments during high flow rates, and 1-mL increments during low flow rates, were used.

After each experiment the cores were removed from the cells and weighed, oven dried at 105 °C for 24 h, and weighed again to determine the volumetric water content at the end of the experiment. Together with the cumulative outflow volume and the volume outflow during the initial equilibration (−30 cm), this water content was subsequently used to calculate saturated and initial water contents. In the parameter optimization procedure, cumulative outflow with these water content values determined the time sequence of water content data corresponding to the simulated soil water pressure values within the soil core. In all optimizations, the mass balance error was mostly <1%. Additionally, in Exp. A, cumulative outflow volume and initial water content were used to calculate the equilibrium water contents. The saturated hydraulic conductivities of each ceramic plate were determined from cumulative outflow volume from modified pressure cells filled with water when subjected to an arbitrarily chosen pneumatic pressure head of 400 cm.

The parameters α, n, θ_r, and K_s were optimized by numerical inversion using MULSTP by either including cumulative outflow data as a function of time alone, or by supplementing it with soil water pressure head as a function of time in the objective function. The hydraulic conductivity of the plate and θ_s were considered to be fixed. To test uniqueness, each inversion problem was run on the computer three times, with the second and third runs using different initial parameter estimates obtained by automatic perturbation of the initial estimates in the first run. Initial parameter values for the Yolo, Panoche, and Hanford soils were chosen to closely approximate available estimates for medium-textured soils (Kool et al., 1985a), while those for the Oso–Flaco sand were adjusted to reflect the coarser particle-size distribution.

Soil water retention curves were generated using final estimates of the parameters with a combination of highest r^2 and lowest sums of squares between fitted and observed data. The curves generated with parameters obtained with cumulative data alone were compared with those obtained with outflow data supplemented with soil water pressure head data. The comparison was done by calculating the root mean squared error from optimized and measured volumetric water content

Table 1. Particle-size analyses and bulk densities of investigated soils.

Soil	Sand	Silt	Clay	Bulk Density
		%		g cm^{-3}
Yolo silt loam	23.0	55.5	22.5	1.17
Panoche loam	37.5	42.5	20.0	1.22
Hanford sandy loam	65.0	23.8	11.3	1.45
Oso–Flaco fine sand	100.0	0.0	0.0	1.53

values from Exp. A:

$$\mathrm{RMSE} = \left\{1/D \sum_{i=1}^{D} [\theta_o(h_o) - \theta_c(h_o)]^2\right\}^{0.5} \quad [7]$$

where RMSE is the root mean squared error, and D is the number of data points in Exp. A.

RESULTS AND DISCUSSION

In both the pressure and suction experiments, the duration of one-step and multistep experiments differed considerably. One-step experiments took between 1 and 4 d while multistep experiments took between 3 and 7 d. Despite the fact that we used deaerated water to saturate the soil samples, air bubbles accumulated under the ceramic plate by diffusion of dissolved air at a soil water pressure head of about −200 cm after a period of ≈6 h, with considerably more air bubbles in the suction than in the pressure experiments (see also Klute, 1986). Since the experimental setup allows for air bubble removal, the accumulating air bubbles were removed promptly to avoid errors in outflow measurements. Air accumulation in the tensiometer started at that soil water pressure head in the suction experiment only. Figures 2 through 5 show the optimized soil water retention curves for the Yolo silt loam, Panoche loam, Hanford sandy loam, and Oso–Flaco fine sand, respectively. Each figure shows the optimized curves for four experiments: one-step transient outflow under pressure (Exp. A), one-step transient outflow under suction (Exp. B); multistep transient outflow under pressure (Exp. C), and multistep transient outflow under suction (Exp. D). Also shown in these figures are the independently measured soil water retention data points obtained from Exp. A. Figure 6 is a comparison between measured soil water retention data determined from desorption under pressure and suction.

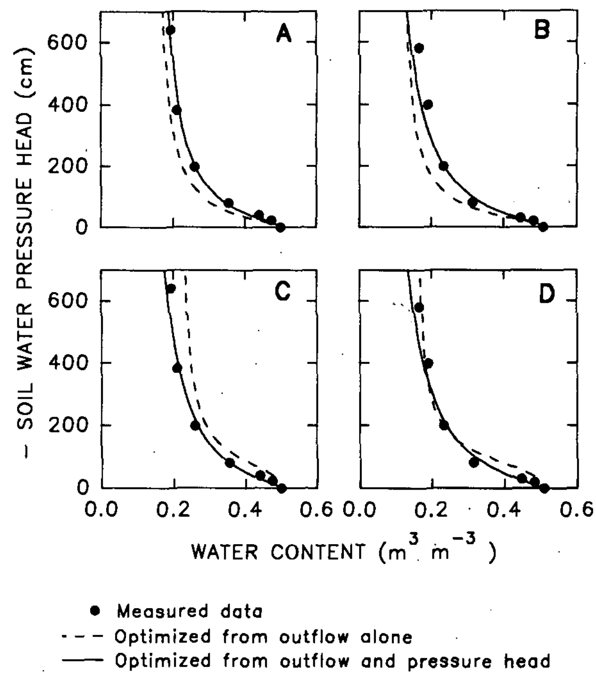

Fig. 3. Soil water retention curves for Panoche loam: (A) pressure one-step; (B) suction one-step; (C) pressure multistep; and (D) suction multistep.

In general, the results in Fig. 2 through 5 show that the independent soil water retention data are better described by parameter optimization including both outflow and soil water pressure head data than cumulative outflow data alone. Shown in Fig. 2 are soil water re-

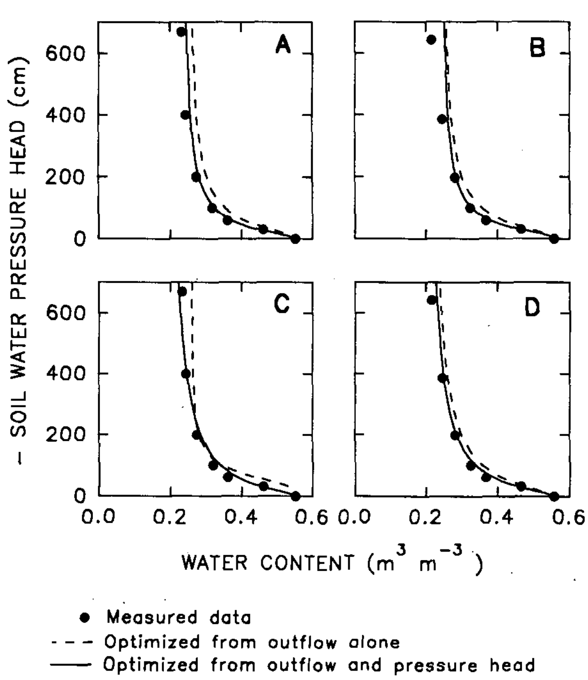

Fig. 2. Soil water retention curves for Yolo silt loam: (A) pressure one-step; (B) suction one-step; (C) pressure multistep; and (D) suction multistep.

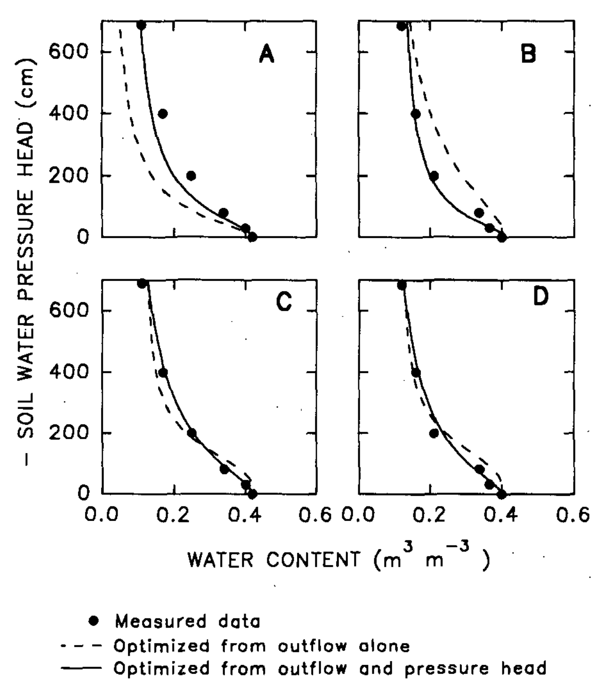

Fig. 4. Soil water retention curves for Hanford sandy loam: (A) pressure one-step; (B) suction one-step; (C) pressure multistep; (D) suction multistep.

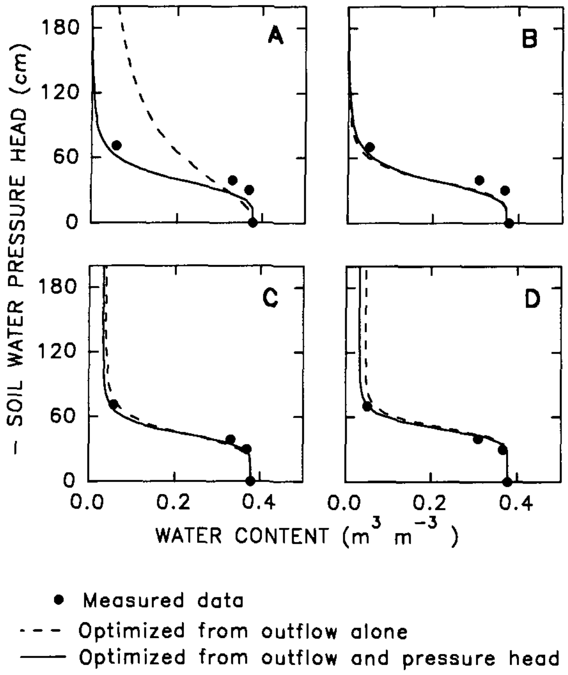

Fig. 5. Soil water retention curves for Oso–Flaco fine sand: (A) pressure one-step; (B) suction one-step; (C) pressure multistep; and (D) suction multistep.

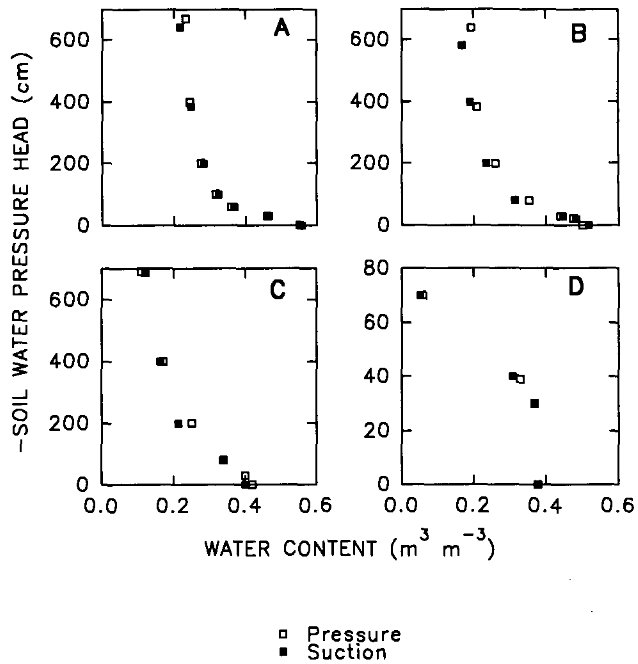

Fig. 6. Soil water retention data by pressure and suction desorption: (A) Yolo silt loam; (B) Panoche loam; (C) Hanford sandy loam; and (D) Oso–Flaco fine sand.

tention curves for the Yolo silt loam. The difference between the two curves are small in all experiments, but the curves obtained from both outflow and soil water pressure head is better in all cases except in the one-step suction experiment. This could have been due to an undetected leak in the pressure cell at the later stages of the experiment. The results for Panoche loam (Fig. 3) are similar to that of the Yolo soil. A relatively good agreement between measured soil water retention data and those optimized from outflow and soil water pressure head data was obtained, except for slight deviations at the dry end of the suction experiments. This deviation could have been minimized if we had assigned larger weights (W in Eq. [6]) to the soil water pressure head data at low water contents. Results for the Hanford sandy loam are presented in Fig. 4. It is apparent from these curves that the multistep experiments (Fig. 4C and 4D) match the equilibrium data better than the one-step experiments (Fig. 4A and 4B). Also, we note that, if cumulative outflow is used alone, multistep experiments gave better results than one-step optimization. Comparison of the optimized soil water retention curve for the Oso–Flaco fine sand is shown in Fig. 5. The results show the advantage of using the multistep methodology if cumulative outflow alone is measured. Good results are obtained as well with one-step experiments if soil water pressure head is measured simultaneously with outflow. The measured soil water retention data in Fig. 5 illustrate another important point. Although the last pressure or suction increment was 200 cm, the measured pseudo-equilibrium soil water pressure head in the center of the soil core was never below -70 cm. At the corresponding water content, the hydraulic conductivity was too low for drainage to occur, thereby preventing the soil from coming to equilibrium. Hence, care must be taken to avoid significant errors in the soil water retention curve if no soil water pressure head measurements are taken. Although not as significant, differences between observed and imposed pressure or suction increments were observed in the other soil types at the 700-cm pressure–suction head increment as well.

A quantitative comparison between the curves is summarized in Table 2, where values of the RMSE (Eq. [7]) for all experiments are tabulated. The smaller the RMSE value (m^3 m^{-3}) the better the optimized curve approximates the equilibrium soil water retention data. Overall, the RMSE values for all curves optimized with both measured outflow and pressure head data are 25 to 50% of

Table 2. Root mean squared error for all soils and experiments.

Experiment	Root mean squared error: Outflow alone	Root mean squared error: Outflow and pressure head
	m^3 m^{-3}	
Yolo		
One-step pressure	0.0319	0.0101
One-step suction	0.0318	0.0162
Multistep pressure	0.0429	0.0143
Multistep suction	0.0237	0.0118
Panoche		
One-step pressure	0.0341	0.0099
One-step suction	0.0819	0.0136
Multistep pressure	0.0412	0.0102
Multistep suction	0.0247	0.0135
Hanford		
One-step pressure	0.0748	0.0284
One-step suction	0.0437	0.0220
Multistep pressure	0.0266	0.0106
Multistep suction	0.0264	0.0137
Oso–Flaco		
One-step pressure	0.0740	0.0693
One-step suction	0.0295	0.0688
Multistep pressure	0.0662	0.0201
Multistep suction	0.0218	0.0168

Table 3. Optimized parameters with outflow only (Q), and both outflow and soil water pressure head (h) in objective function.

	One-step pressure		One-step suction		Multistep pressure		Multistep suction	
Parameter†	Q‡	Q and h§	Q	Q and h	Q	Q and h	Q	Q and h
				Yolo silt loam				
α, cm^{-1}	0.0266	0.0360	0.0250	0.0319	0.0169	0.0367	0.0291	0.0358
n	1.980	1.905	2.003	2.072	3.094	1.572	1.672	1.645
θ_r, m^3 m^{-3}	0.245	0.228	0.240	0.242	0.258	0.161	0.190	0.182
K_s, cm h^{-1}	3.605	18.125	3.461	15.667	0.074	5.876	1.937	5.896
θ_s, m^3 m^{-3}¶	0.552	0.552	0.558	0.558	0.552	0.552	0.558	0.558
				Panoche loam				
α	0.0316	0.0247	0.0285	0.0239	0.0120	0.0228	0.0112	0.0207
n	1.761	1.605	1.937	1.488	2.529	1.459	2.906	1.488
θ_r	0.137	0.119	0.108	0.003	0.223	0.051	0.163	0.002
K_s	7.381	3.148	7.114	1.768	0.397	1.405	0.416	1.817
θ_s	0.500	0.500	0.517	0.517	0.500	0.500	0.517	0.517
				Hanford sandy loam				
α	0.0168	0.0131	0.0055	0.0173	0.0072	0.0088	0.0065	0.0103
n	1.871	1.880	1.911	1.878	3.076	1.685	3.053	1.598
θ_r	0.005	0.055	0.040	0.102	0.117	0.010	0.114	0.005
K_s	2.689	2.592	0.554	6.870	0.232	0.893	0.134	1.138
θ_s	0.420	0.420	0.400	0.400	0.420	0.420	0.400	0.400
				Oso–Flaco fine sand				
α	0.0219	0.0262	0.0260	0.0263	0.0229	0.0226	0.0197	0.0201
n	2.241	4.840	5.352	4.828	6.063	7.339	9.213	9.326
θ_r	0.003	0.000	0.000	0.000	0.039	0.032	0.046	0.033
K_s	13.776	4.842	11.059	13.656	12.822	2.214	5.279	2.289
θ_s	0.378	0.378	0.378	0.378	0.378	0.378	0.378	0.378

† Parameters defined in Eq. [3], [4], and [5].
‡ Cumulative outflow only in objective function.
§ Cumulative outflow and soil water pressure head in objective function.
¶ θ_s fixed in all the optimizations in the four soils.

the RMSE values for those optimized with outflow alone, except for the Oso–Flaco fine sand, where RMSE values are only slightly smaller if pressure head is measured. Table 2 also shows that multistep experiments yielded lower RMSE values than one-step experiments except for the pressure experiments of Yolo silt loam and Panoche loam. While measuring outflow and soil water pressure head in a draining soil sample using multiple steps, retention points along the soil water retention curve are better defined than in a one-step experiment. As also shown by Toorman et al. (1992), parameter sensitivity and uniqueness improves as more information toward estimation of the soil water retention is included. Differences in RMSE values between the one-step pressure and one-step suction, or multistep pressure and multistep suction, are very small. In either case, multistep experiments with soil water pressure head data gave excellent results.

Final parameter values for each soil and experiment are listed in Table 3. Only those parameter values for the run with a combination of highest r^2 and lowest sums of squares between fitted and observed data are reported. We found that optimization with different sets of initial parameter values yielded nearly identical final parameter estimates in most cases when both outflow and soil water pressure head were included in the objective function. Thus, if optimization is carried out using both outflow and soil water pressure head data, the obtained parameter values are seemingly unique for the experimental conditions under consideration. Our results agree with the response surface analyses reported by Toorman et al. (1992). Although their data sets were numerically generated, they found that the combination of outflow and pressure head measurements increased the sensitivity of the parameter estimation procedure with the one-step outflow method, thereby allowing convergence to a unique solution. Uniqueness problems have been a major drawback of the inverse solution technique. The approach presented here minimizes this problem for the experimental conditions exercised. To illustrate the above point, the coefficients of variation computed from the three different combinations of initial parameter values for all optimized pressure experiments are presented in Table 4. The coefficients of variation for all four estimated parameters are significantly smaller if both outflow and pressure head data are included in the objective function. Inasmuch as we were able to obtain an excellent description of the soil water retention curves by the inverse solution technique, it must be emphasized that the results are only applicable to the soil water pressure head range of the experiment, and that extrapolation beyond this range is speculation only. Toorman et al. (1992) reported results with tensiometers placed at various distances from the porous plate, and concluded that pressure head measurements should be taken at some distance away from the porous plate. The experimental setup (Fig. 1) used in this study allows the tensiometer to slide up and down, thereby allowing the depth of measurement to vary. All pressure head measurements in this study were made at 3 cm above the ceramic plate.

Differences between soil water retention data obtained by equilibrium stepwise desorption under pressure and suction for the four soils in this study are compared in Fig. 6. Though the differences between the two methods

Table 4. Coefficients of variation for parameters of the pressure experiment computed from three different combinations of initial parameter estimates.

	Coefficients of variation			
	One-step experiments		Multistep experiments	
Parameter†	Outflow alone	Outflow and pressure head	Outflow alone	Outflow and pressure head
	%			
	Yolo silt loam			
α, cm^{-1}	12.01	0.00	1.44	0.04
n	3.15	0.01	1.93	0.00
θ_r, m^3 m^{-3}	4.74	0.07	1.16	0.00
K_{sat}, cm h^{-1}	23.26	0.04	36.96	0.18
	Panoche loam			
α	13.03	0.06	22.58	0.00
n	3.37	0.01	6.88	0.02
θ_r	7.91	0.13	22.05	0.10
K_{sat}	20.44	0.13	14.54	0.07
	Hanford sandy loam			
α	14.68	3.97	34.45	0.70
n	1.22	0.30	18.01	0.18
θ_r	36.00	7.74	42.13	0.00
K_{sat}	30.47	11.81	48.41	1.81
	Oso–Flaco fine sand			
α	8.23	13.42	0.54	0.07
n	0.74	7.45	5.14	0.82
θ_r	18.86	40.45	45.38	5.48
K_{sat}	18.41	36.68	12.60	3.42

† Parameters defined in Eq. [3], [4], and [5]

were not as pronounced as presented by Chahal and Yong (1965), the soil water retention data obtained by suction desorption are slightly below those obtained by pressure. This is particularly visible for the Panoche loam (Fig. 6b). As hypothesized by Chahal and Yong (1965), if pressure were released in the pressure experiments, soil water pressure would increase due to a volume increase of entrapped air. The small difference between the two methods in this study may be caused by small trapped air volumes in our soils samples.

All analyses considered in our study consisted of comparison of soil water retention curves only, while the inverse solution yields unsaturated hydraulic conductivity curves as well. The difficulty in independent measurement of the hydraulic conductivity data has precluded the comparison between independently measured and optimized hydraulic conductivity curves. Because all experiments were started when the soil was slightly unsaturated, an optimized K_s value has no physical meaning and becomes a fitting parameter that should not be compared with measured saturated conductivity. If we consider, however, the soil core with the porous plate as a two-layer system for which flow is perfectly described by Richards' equation, then the optimized parameters should represent the unsaturated hydraulic conductivity function as well if it is assumed that the van Genuchten (1980) analytical expressions adequately describe the soil hydraulic functions.

SUMMARY AND CONCLUSIONS

We showed that computer optimization of soil water retention curves by the inverse solution technique using transient outflow experiments is greatly improved when cumulative transient outflow data are supplemented with simultaneously measured soil water pressure head data. This is a significant finding since outflow experiments allow considerable freedom in the choice of initial and boundary conditions and yet can be kept simple enough to be applicable to different soils. Moreover, considerable time can be saved since equilibrium soil water retention data are not needed. With inclusion of soil water pressure head data in the optimization, both one-step and multistep experiments gave excellent results, and the optimized curves agreed well with the independently measured $\theta(h)$ data. Considerably more data, however, are collected in multistep than in one-step experiments, and multistep experiments take twice as long to perform. Hence the one-step experiment is an attractive alternative. Furthermore, optimization with both measured outflow and soil water pressure head data resulted in almost identical final parameter estimates for each run, irrespective of the initial estimates. Thus, we believe that the problem of uniqueness is reduced when this approach is used. Differences in measured and optimized retention data between pressure and suction experiments were small.

We must also point out that the optimized soil hydraulic functions as determined from soil cores do not necessarily represent in situ soil water behavior. Moreover, the maximum clay content for the investigated soils in this study was 22.5%. Experience with the outflow technique for soils with higher clay contents has shown that the range in soil water pressure achieved with the outflow technique is too small to obtain significant drainage. Also, to make comparison between experiments possible, we used disturbed soils. Undisturbed soil cores may have significant soil variability within the sample, thereby affecting the sensitivity of outflow and the single pressure measurement on the optimization procedure. We analyzed draining soil cores only. We anticipate that hysteretic behavior can be accounted for by including additional parameters, as shown by Kool and Parker (1988).

Previous studies involving measurement of soil water pressure head inside a laboratory soil core have used tensiometers installed horizontally (Boels et al., 1978; Hudson et al., 1991). Our study indicates that soil water pressure head measurement in soil cores can be done accurately with a tensiometer installed vertically, without special arrangements. We have not presented results for hydraulic conductivity functions due to lack of validation data, but a comparison of multistep with independent unsaturated hydraulic conductivity data is ongoing.

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