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Estimation of in situ unsaturated soil hydraulic functions from scaled cumulative drainage data

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Abstract. Simulation of water flow and transport processes in soils rely on field representative soil hydraulic functions. The linear variability concept in combination with the inverse technique was used to estimate in situ soil hydraulic properties in a 32-ha field. Measured cumulative drainage curves were scaled yielding scaling factors. Subsequently, the drainage and moisture content distribution of the scaled reference profile were input to a numerical model to optimize the soil water retention and hydraulic conductivity curves for the reference soil profile by inverse solution of the scaled Richards equation. Field hydraulic functions for each location were computed from the reference curves and scaling factors. In addition, undisturbed soil cores taken from 0.3-m and 0.6-m depths at 44 locations were used to determine soil texture, and soil water retention and hydraulic conductivity curves in the laboratory using the multistep outflow technique. These hydraulic functions were scaled using the simultaneous scaling technique. The reference field hydraulic functions compared well with those determined from the soil cores taken from the 0.6-m depth. In situ saturated hydraulic conductivity variability was one order of magnitude less than that of the soil cores.

Introduction

Numerical models are widely used in the solution of problems of water movement in the vadose zone and in the determination of transport rate of contaminants toward groundwater. Accurate simulation of these transport processes is dependent upon availability of representative soil hydraulic functions: the hydraulic conductivity K as a function of soil water pressure head (h) or soil water content (θ), and the soil water retention function $\theta(h)$. However, these nonlinear functions are difficult to obtain directly, and measurement techniques are time consuming. Usually, many soil samples are required to characterize spatial variability in large field settings. In recent years, there has been a lot of interest in the inverse solution of the flow process as a quick and cost effective method to obtain estimates of the parameters of the hydraulic functions.

Since its inception by *Miller and Miller* [1956], the concept of scaling has been modified several times. *Warrick et al.* [1977] eliminated the original *Miller and Miller* [1956] assumption of identical porosities, thus extending the use of similar soils to soils with different internal geometries. The concept of functional similarity was introduced by *Simmons et al.* [1979]. More recently, *Vogel et al.* [1991] incorporated their scaling relations in numerical modeling of water movement in soil profiles. Other workers have also used scaling for modeling soil water [*Peck et al.*, 1977; *Warrick and Amoozegar*, 1979; *Sharma et al.*, 1980; *Youngs and Price*, 1981; *Hopmans and Stricker*, 1989; *Ünlü et al.*, 1990]. *Tillotson and Nielsen* [1984] and *Hopmans* [1987] reviewed a variety of methods to obtain scaling factors.

Over the past few years, workers in hydrologic science and related fields have used the inverse solution technique to estimate soil hydraulic properties. Parameter estimation involves the numerical solution by inversion of the flow process. First, the soil hydraulic properties are assumed to be described by an analytical model with unknown parameter values. The unknown parameters are then estimated by minimization of the square of deviations between measured and predicted flow variables. The technique was applied to unsaturated one-step laboratory outflow experiments by *Parker et al.* [1985]. *Van Dam et al.* [1990] modified the one-step technique by increasing pressure in several steps (multistep). *Eching and Hopmans* [1993] showed how concurrent measurement of soil water pressure head and cumulative volume outflow from various one-step and multistep experiments improved the inverse solution. *Dane and Hruska* [1983], *Kool et al.* [1987], and *Sisson and van Genuchten* [1991] determined soil hydraulic properties by inversion of in situ water content data from vertical drainage. *Kool and Parker* [1988] used the technique in a flow process consisting of ponded infiltration, followed by gravity drainage with evaporation at the soil surface. Researchers have also coupled numerical inverse problems such as heat or mass transport with unsaturated or saturated flow through a hydrogeologic setting [*Carrera*, 1987; *Mishra and Parker*, 1989; *Sun and Yeh*, 1990].

There is a need to combine the scaling and the inverse solution techniques in order to determine soil hydraulic functions quickly and to describe their spatial distribution. The purpose of this study therefore was to combine the linear variability scaling technique introduced by *Vogel et al.* [1991] with the inverse solution technique to determine in situ soil hydraulic properties from draining soil profiles. Although several field methods already exist for in situ determination of soil hydraulic properties [*Davidson et al.*, 1969; *Libardi et al.*, 1980; *Chong et al.*, 1981; *Jones and

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Wagenet, 1984; *Sisson and van Genuchten*, 1991], this study differs from existing methods in that it combines the quickness of the parameter estimation technique with the scaling capability. *Shouse et al.* [1992] used such an approach to successfully estimate unsaturated hydraulic properties of vertical heterogeneous soils. Our method of scaling differs from their study in that we scale the flow process itself so that scale factors relate to the scaled Richards equation. Moreover, the present techniques emphasizes soil heterogeneity in the horizontal direction. The method allows the determination of soil spatial variability in terms of scaling factors, thus allowing the number of parameters required to describe soil hydraulic functions in a spatial variable field to be reduced. Data acquisition and computer data analysis are also reduced.

Theory

Linear Variability Concept

A detailed description of the linear variability scaling concept has been given by *Vogel et al.* [1991]. Assuming global soil variability to be characterized by one-dimensional soil profiles, the soil-profile-representative hydraulic properties can be expressed in terms of a linear transformation

$$\begin{aligned} K(i,h) &= \gamma_K(i)K^*(h^*) \\ \theta(i,h) &= \theta_r(i) + \gamma_\theta(i)[\theta^*(h^*) - \theta_r^*] \qquad (1) \\ h(i) &= \gamma_h(i)h^* \end{aligned}$$

where i is a position vector, $K(h)$ and $\theta(h)$ are the soil hydraulic functions representing an individual homogeneous profile at location i, and $K^*(h^*)$ and $\theta^*(h^*)$ are space invariant soil hydraulic functions of a reference hypothetical soil profile. The scaling factors γ_K, γ_θ, and γ_h relate hydraulic conductivity, moisture content, and pressure head of each profile i to the reference soil profile, with θ_r denoting residual water content. As is discussed by *Vogel et al.* [1991], scaling factors and reference profile hydraulic functions are defined such that the arithmetic means of γ_K and γ_θ are equal to 1. Successful application of the linear variability concept is constraint to the requirement that γ_h is one for any finite soil domain.

One-dimensional vertical flow of water in the individual soil profiles as well as in the reference profile is described by Richards equation with t denoting time (T) and z representing depth (L), being defined as positive downward

$$\partial\theta/\partial t = \partial/\partial z[K(\partial h/\partial z - 1)] \qquad (2)$$

The assumptions underlining the scaling technique are that the initial pressure head profile is identical for all profiles and that pressure head boundary values are equal for corresponding boundary points of the different profiles. Subject to the assumptions given above, (2) is solved for the reference profile. We assumed the soil hydraulic function $K(\theta)$ and $h(\theta)$ to be described by the *van Genuchten* [1980] expressions

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = S_e = [1 + |\alpha h|^n]^{-m} \qquad (3)$$

$$K(\theta) = K_s S_e^l[1 - (1 - S_e^{1/m})^m]^2 \qquad (4)$$

where S_e is the effective saturation ($0 \le S_e \le 1$), $\theta_r(L^3L^{-3})$ and $\theta_s(L^3L^{-3})$ are the residual and saturated water contents, respectively, $K_s(LT^{-1})$ is the saturated hydraulic conductivity, and $\alpha(L^{-1})$, n, m, and l are empirical parameters; l was assumed to be 0.5, and $m = 1 - 1/n$.

The scaling relations in (1) are then applied to compute Darcian flux $q(LT^{-1})$ of location i in terms of the flux of the reference profile, q^*:

$$q(t, i) = \gamma_K(i)q^*(t^*) \qquad (5)$$

where reference time t^* is related to time for any other profile $t(i)$ by

$$t^* = \frac{\gamma_K(i)}{\gamma_\theta(i)}\, t(i) \qquad (6)$$

Hence fluxes for each profile i are linearly proportional to the corresponding fluxes in the reference profile, with the scaling factor γ_K being the proportionality factor. Integrating (5) with respect to time as defined in (6), we obtain the scaling relationship for cumulative drainage $Q(L)$:

$$Q(t, i) = \gamma_\theta(i)Q^*(t^*) \qquad (7)$$

Scaling Drainage Curves

Having available a set of measured cumulative drainage curves as a function of time $Q(t)$, we can relate reference drainage Q^* to drainage Q and drainage rate q^* to q at any measured location i by

$$Q(t, i) = \gamma_Q(i)Q^*(t^*) \qquad (8)$$

$$q(t, i) = \gamma_q(i)q^*(t^*) \qquad (9)$$

and time t to t^* by

$$t(i) = \gamma_t(i)t^* \qquad (10)$$

where γ_Q and γ_t are scale factors for cumulative drainage and time, respectively, and the other symbols have been defined before. Combining (7) with (8), (5) with (9), and (6) with (10), it follows that

$$\gamma_q(i) = \gamma_K(i) \qquad (11a)$$

$$\gamma_Q(i) = \gamma_\theta(i) \qquad (11b)$$

$$\gamma_t(i) = \gamma_\theta(i)/\gamma_K(i) \qquad (11c)$$

The relationships in (11) imply that we can relate the variability in soil hydraulic functions to the variability in cumulative drainage of the same locations and that we can use those relationships to calculate one from the other if the drainage characteristics of the reference profile are known. In this paper we apply the inverse solution technique to obtain $\theta^*(h^*)$ and $K^*(\theta^*)$ using the initial and boundary conditions of the reference soil profile. The upper boundary was zero flux, and the bottom boundary was the drainage rate. Water content distribution at time zero was used as initial condition. These are obtained from water content profiles as measured with a neutron probe. We then estimate the soil hydraulic functions and their spatial variability representative for the field from the reference profile hydraulic functions and the scaling factors γ_Q and γ_t of measured cumulative drainage.

Calculation of Scaling Factors

In our analysis we assume that drainage flux rate q is described by a simple exponential function [*Belmans et al.*, 1983] of the form

$$q = a \exp(-bt) \tag{12}$$

where a and b are fitting parameters and t is time. Among various empirical relationships investigated, the best fit to measured drainage data was obtained using this exponential relationship. Integrating (12) yields the cumulative drainage curve $Q(t)$ for any profile i and $Q^*(t^*)$ for the reference curve,

$$Q(i) = \frac{a(i)}{b(i)} \{1 - \exp[-b(i)t(i)]\} \tag{13}$$

$$Q^* = \frac{a^*}{b^*} [1 - \exp(-b^* t^*)] \tag{14}$$

Combining (8) and (10) with (13) results in

$$Q^* = \frac{a(i)}{b(i)\gamma_Q(i)} \{1 - \exp[-b(i)\gamma_t(i)t^*]\} \tag{15}$$

From (14) and (15) it follows that

$$\gamma_t(i) = b^*/b(i) \tag{16}$$

$$\gamma_Q(i) = \frac{a(i)}{b(i)} \frac{b^*}{a^*} \tag{17}$$

Furthermore, it can be shown that

$$\gamma_q(i) = a(i)/a^* \tag{18}$$

In summary, we fit (13) to the measured cumulative drainage data at each location i to obtain a_i and b_i. By using the arithmetic mean of the fitting parameters a and b($\bar{a}$ and $\bar{b}$) as initial guess values for a^* and b^* of the reference profile drainage curve, we compute $\gamma_t(i)$, $\gamma_Q(i)$ and $\gamma_q(i)$ for each location by applying (16), (17), and (18). To satisfy relationships in (11a) and (11b), the arithmetic mean of γ_Q and γ_q must be equal to 1. If the arithmetic mean is not equal to 1, γ_Q and γ_q are normalized by dividing these individual scaling factors by their respective arithmetic means ($\bar{\gamma}_Q$ and $\bar{\gamma}_q$) [*Vogel et al.*, 1991]. After normalization, fitting parameters for the reference drainage curve are calculated from

$$a^* = \bar{\gamma}_q \bar{a} \tag{19}$$

$$b^* = \frac{a^*}{\bar{\gamma}_Q} \frac{\bar{b}}{\bar{a}} \tag{20}$$

The relations between the scaling factors of the drainage curves and those of the soil hydraulic function are given in (11). Using $\theta^*(h^*)$ and $K^*(\theta^*)$ computed from the inverse solution, the soil hydraulic functions of all measurement locations can be determined from (1).

Materials and Methods

An experiment was set up in a 32-ha furrow-irrigated field in the west side of the San Joaquin Valley, California. The soil is of the Panoche series, having very deep and well drained uniform profiles with a wide range of textures. Soil texture varied from a silty loam and sandy clay loam on the south east side of the field to a loamy sand and sandy loam with patches of silty clay, clay loam, and silty clay in the rest of the field. The soils have uniform profiles with no significant layering. The field was instrumented with 44 neutron access tubes to a depth of 2.1 m spaced 75 m apart along the furrow bed (11 locations) and 100 m across (4 rows). A subsurface drainage system is installed throughout the field at the 2.5-m depth. The field received 0.3 m of water during a preplant irrigation in late January. Water was applied to the field from an irrigation ditch at the head end of the field through polyvinyl chloride siphons. Soil water content was measured with a neutron probe at 0.15-m intervals from 0.15 to 0.9 m and at 0.3-m intervals from 0.9- to 2.1-m depth. Moisture contents were measured 6 times thereafter for a duration of 125 days. The starting time for drainage (time = 0) corresponded with the time at which the soil profile water content was maximum. This occurred 4 days after the beginning and 1 day after the end of irrigation. The 125-day measurement period extended from January 24, to May 30, 1990. There was a water table in the field below the 2.1-m soil depth during this measurement period. The field was seeded in cotton in late April. Rainfall (0.037 m) and estimated evapotranspiration (0.035 m) in April and May were approximately equal. Evapotranspiration was calculated from crop coefficient and reference evapotranspiration data measured at a weather station 3 km from the field. Rainfall was also measured at the station. Historical [*California Department of Water Resources*, 1986] data from Class A evaporation pan during February and March (0.028 m) was also approximately equal to measured rainfall for the same period (0.033 m). Hence net changes in moisture content during the 125-day observation period were attributed to drainage only.

Cumulative drainage as a function of time at each location was calculated from the change in storage as measured by the neutron probe. A curve of the form in (13) was fitted through the cumulative drainage data for each location i using Powell's minimization algorithm [*Clausnitzer et al.*, 1990]. This yielded the drainage curve parameters $a(i)$ and $b(i)$ for each of the 44 locations. A reference profile cumulative drainage curve for the entire field with parameters a^* and b^* was then obtained by using the scaling technique described. Reference profile moisture content distributions for the entire field to be used in the subsequent inverse modeling was obtained by arithmetic averaging the moisture content distribution at each measurement depth and time.

Undisturbed soil cores were taken from the 0.3- and 0.6-m soil depth at the same 44 locations. These core samples were brought to the laboratory for measurement of soil hydraulic functions and soil textural analysis. Measurement locations in the field were grouped into clays (6 locations), loams (12 locations), and sands (26 locations) based on textural analysis of the undisturbed soil cores. Reference profile drainage curves for each of these soil textural groups were also determined. Parameters for the reference profile drainage parameters and soil profile moisture content distributions for the clays, loams, and sands were obtained from 6, 12, and 26 observation sites, respectively.

Laboratory Method

The 6-cm-long, 8.25-cm-wide undisturbed soil cores taken from each of the 44 locations were taken to the laboratory and assembled in Tempe pressure cells with 0.57-cm-thick and 1.0-m air entry ceramic plates. The samples were then saturated by wetting from the bottom with 0.01 M $CaCl_2$ solution. Multistep outflow experiments [*van Dam et al.*, 1990; *Eching and Hopmans*, 1993] with pressure steps of 0.5, 1.0, 2.05, 4.9, and 10 m were carried out on the samples. The hydraulic parameters α, n, θ_r, and K_s for each undisturbed soil sample were obtained by the inverse solution of the multistep outflow experiments with the measured θ_s for each sample fixed. The objective function (OF) in the optimization involved the minimization of the squared deviation between measured (Q_m) and predicted (Q_p) cumulative outflow volumes (L^3):

$$OF = \sum_{i=1}^{N} [(Q_m(t_i) - Q_p(t_i, b))]^2 \qquad (21)$$

where $\{b\}$ is a vector containing the optimized parameters $\{\theta_r, \alpha, n, \text{and } K_s\}$. In our experiments we measured θ_s and assumed the value of l to be equal to 0.5. N is the number of cumulative outflow volumes, and t is time at which measurement was made. Laboratory reference soil water retention and hydraulic conductivity curves for the combined depths of each soil group and for the whole field were obtained by using the simultaneous scaling procedure [*Clausnitzer et al.*, 1992], where soil water retention and hydraulic data are scaled simultaneously using the scaling relations as in the work by *Warrick et al.* [1977]. The procedure results in scaled reference curves for both the water retention and hydraulic conductivity data as well as a single set of scaling factors characterizing the spatial variability of the soil hydraulic functions. Simultaneous scaling was used rather than the linear scaling of *Vogel et al.* [1991] to characterize the variability of the laboratory data. The several changes in the pressure head boundary condition (multistep) during the laboratory experiment violated the assumptions of the linear variability concept. Additional separate reference curves were computed for the 0.3- and 0.6-m soil depths for the whole field and for the three individual soil groups.

Field Method 1

The van Genuchten parameters α, n, and θ_r [*van Genuchten*, 1980] and the saturated hydraulic conductivity for reference soil hydraulic functions for the field and each soil textural group were optimized from reference cumulative drainage and reference moisture content distributions by the inverse solution of (2). These functions then represent the whole 2.1-m soil profile. The objective function (OF) minimized in the optimization was the squared deviation between measured (θ_m) and predicted water contents (θ_p) for the reference soil profile

$$OF = \sum_{i=1}^{N} [\theta_m(t_i) - \theta_p(t_i)]^2 \qquad (22)$$

where i is measurement index and N is the number of soil moisture measurements during the drainage period. Reference soil profile drainage rate was used as a bottom flux boundary condition. The highest water content measured at each field location after irrigation was considered to be the saturated water content at time zero. The saturated water content values (θ_s) for the reference hydraulic functions were obtained by arithmetic averaging of field measured individual θ_s values and were considered to be fixed.

As initial condition we used the reference profile volumetric water content profile at time zero with a water table at the 2.1-m depth. Although the linear scaling procedure scales $\theta - \theta_r$, we used the arithmetic average of the soil water content distribution of the individual locations as the reference profile soil water content distribution, since in the absence of individual θ_r it was impossible to obtain ($\theta^* - \theta_r^*$) beforehand. We assumed a zero flux boundary condition at the soil surface. Individual retention and conductivity functions were subsequently computed from the reference hydraulic functions and the scaling factors in (1), determined from (16) and (18).

Field Method 2

The laboratory soil hydraulic data were obtained by individual optimization of each soil sample, followed by simultaneous scaling, whereas the in situ soil hydraulic functions in field method 1 were determined from optimization of the reference drainage soil profile, with variability expressed by the scaling factors of drainage curves obtained before optimization. Field method 1 is proposed because it saves considerable computing time, as the number of optimizations for the whole field is reduced from 44 to 1. However, one might question the influence of variability calculation on the estimated hydraulic functions results. Therefore in field method 2 we optimized the soil hydraulic functions for each individual drainage curve and computed variability by simultaneous scaling [*Clausnitzer et al.*, 1992] afterward, as was done for the laboratory method.

Cumulative drainage and soil water content distribution at each location of the sand textural group were input to a numerical model to optimize the van Genuchten parameters α, n, and θ_r and the saturated hydraulic conductivity. This soil group is composed of sand, sandy loam, and loamy sand soils. The initial condition for each location was the respective volumetric water content at time zero. Thus the optimization involved the minimization of the squared deviations between measured and predicted water contents for each location. Drainage rate as estimated from measured cumulative drainage was used as a bottom flux boundary condition. As in method 1, we assumed a zero flux boundary condition at the surface and considered the saturated water content (θ_s) fixed. Of the 26 locations in the group, optimization did not converge for 1 location, while 3 locations gave mass balance errors in excess of 18%. Subsequent analysis was applied to the remaining 22 locations. Soil water retention and unsaturated hydraulic conductivity curves for each of these locations were generated from the optimized hydraulic parameters. Reference soil water retention and hydraulic conductivity curves and scaling factors of the hydraulic functions were obtained using the simultaneous scaling procedure described by *Clausnitzer et al.* [1992]. The laboratory reference curves for the combined depths and for the individual depths were compared with the in situ reference curves for the 2.1-m profile from methods 1 and 2.

Results and Discussion

Figure 1a shows measured cumulative drainage (6 measurement times) beyond 2.1 m at the 44 locations as a function of time with the calculated reference curve represented by the solid line. The large scatter reflects the spatial variability of soil hydraulic functions of the 2.1-m deep soil profiles in the 32-ha field. Correlation between cumulative drainage and initial soil water storage (time = 0) was investigated by regression analysis. The coefficient of determination (r^2) was only 0.065, indicating that the variation in cumulative drainage was not caused by variations of initial soil water storage, but by different soil hydraulic functions. Initial profile water storage (in meters) and cumulative drainage (in meters) had coefficients of variation of 0.08 and 0.21, respectively. Equation (15) implies that the individual cumulative drainage curves can be coalesced into a single scaled reference curve. The successes of scaling using the relationship in (15) is shown in Figure 1b, where the individual drainage curves coalesce to a single curve after scaling. Results for the sand group with 22 of the measured 44 locations are shown in Figures 1c and 1d. Linear regression of descaled cumulative drainage data from (8) and (10) with the measured cumulative drainage curves for all 44 locations yielded a coefficient of determination (r^2) of 0.92 and a slope of 0.99. If scaling is perfect, the slope of the regression line between descaled and the original data should be equal to 1, and the data should fall on the 1:1 line. Values of the coefficient of determination (r^2 = 0.92) and slope (0.99) close to 1 demonstrate the successful application of the scaling method. The correlation coefficients and slopes for the three separate soil groups were equally close to 1.

Since the sandy soil constituted 60% of the data, hereinafter we present results for this soil group only. However, we found similar results for the other soil groups and all measurement locations. The optimized reference soil hydraulic parameters for the different methods for the sand group are shown in Table 1. Soil water retention and hydraulic conductivity curves generated with these parameters are shown in Figure 2. As a word of caution, we should point out that the presence of a water table at time zero allowed optimization without soil water pressure head data. Optimization of soil hydraulic functions from water content distribution alone without the existence of a water table needs to be further investigated. Figures 2a and 2d compare the in situ reference profile hydraulic properties with those obtained in the laboratory by combining the 0.3- and 0.6-m-depth data. Comparison of the reference profile field hydraulic functions with the 0.3-m laboratory functions is presented in Figures 2b and 2e, whereas Figures 2c and 2f compare the field functions with the 0.6-m-depth laboratory functions. The agreement between the two field methods is excellent, indicating that the differences in the scaling techniques do not affect the results. Comparison of curves from the entire field with those from the laboratory experiment are similar to those presented in Figure 2.

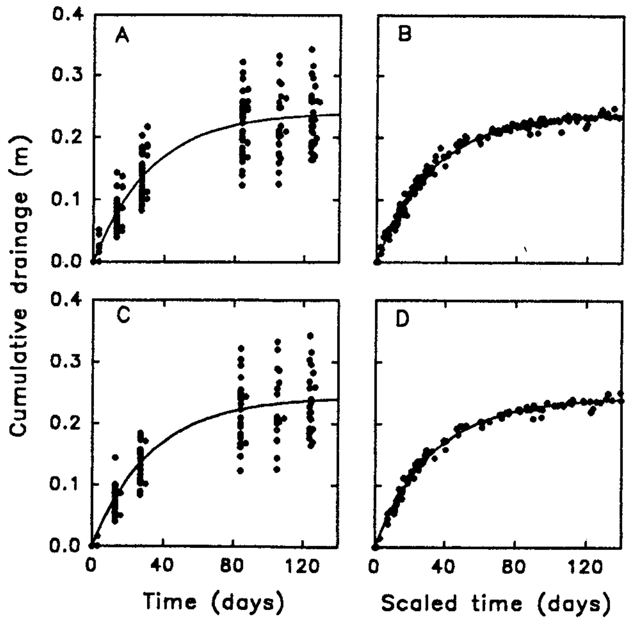

Figure 1. (a) Unscaled and (b) scaled cumulative drainage for the whole field and (c) unscaled and (d) scaled cumulative drainage for group 1. Solid curves represent the reference drainage curve.

Table 1. Optimized Hydraulic Parameters for Field and Laboratory Experiments

	Laboratory			Field	
Parameter	0.3-m Depth	0.6-m Depth	Both Depths	Method 1	Method 2
α, cm^{-1}	0.018	0.040	0.033	0.018	0.024
n	1.838	2.059	1.956	2.757	2.911
θ_r, cm^3 cm^{-3}	0.12	0.09	0.10	0.07	0.09
θ_s,* cm^3 cm^{-3}	0.39	0.41	0.40	0.42	0.42
K_s, cm h^{-1}	1.53	8.50	4.34	0.22	0.17

*Fixed.

In general, we found differences between field reference profile soil water retention curves and laboratory reference soil water retention curves. This is indicative of the fact that while laboratory experiments are more accurate and generally more conveniently measured than in situ experiments, soil properties determined on a small core sample may not be representative of the in situ behavior [*Russo et al.*, 1991]. We have assumed here that $\theta(h)$ and $K(\theta)$ relationships for the small laboratory scale and the large field scale have the same analytic form. The laboratory-determined soil water retention curve for the 0.6-m depth (Figure 2c) approximates the field soil water retention curve well. Combination of the 0.3- and 0.6-m depths also approximated the field data relatively well in the wet range. It seems therefore that to use laboratory core samples to infer in situ soil profile behavior, core samples must be taken from below the plow layer. Soil cores taken from the 0.6-m depth represent the drainage behavior of the 2.1-m soil better than the soil cores taken from the 0.3-m depth.

The hydraulic conductivity obtained from field data is about an order of magnitude less than those obtained from laboratory data (Figures 2d, 2e, and 2f) in the wet water content range but is greater in the dry range. Field hydraulic conductivity in the wet water content range is controlled by the most restricting layer, whereas laboratory cores include both high and low conductive soils. At the end of the experiment, the water content in the field soil was higher than the one in laboratory cores. The laboratory cores were subjected to a pressure of up to 10 m during the outflow

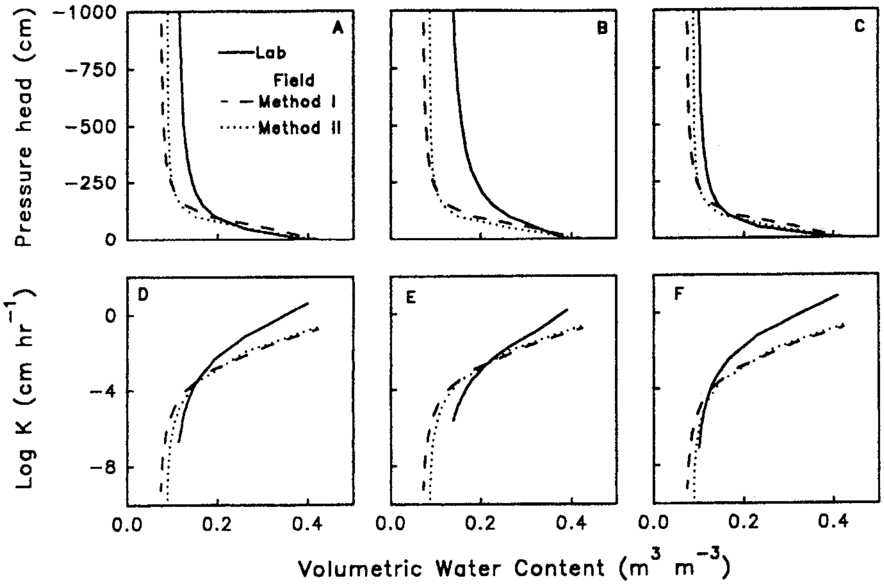

Figure 2. (a)–(c) Comparison of soil water retention and (d)–(f) hydraulic conductivity curves for group 1. Dashed (method 1) and dotted line (method 2) represent in situ curves characterizing whole soil profile. Solid lines represent data from 22 soil core samples (Figures 2a and 2d), 11 samples each from 0.3 m (Figures 2b and 2e), and 0.6 m (Figures 2c and 2f).

experiments, whereas water redistribution in the field occurred by natural drainage only. Consequently, the field optimized hydraulic functions are more representative for the wet water content range. The field hydraulic functions in the dry moisture range are the result of extrapolation beyond experimental data. The field and laboratory saturated hydraulic conductivity were both obtained by parameter optimization. In situ saturated hydraulic conductivities are compared with laboratory-determined saturated conductivity values from the soil samples at the 0.6-m depth in Figure 3. For the most part, estimated field saturated hydraulic conductivity is less than laboratory saturated hydraulic conductivity by an order of magnitude or more. Again, the saturated conductivity of the field profile is determined by the conductivity of the most restricting layer. Figure 3 also shows that the field experimental data exhibit less variability than the laboratory data. The high mean and variability of laboratory K_s compared with field data has also been reported by *Banton* [1993] in a recent study. From Table 2 it is seen that the coefficient of variation of field saturated hydraulic conductivity is smaller than that of the laboratory, and that the difference between the minimum and maximum values is considerably smaller as well. Minimum values are comparable, but maximum values from the laboratory soil cores are much larger (100 versus 0.28–1.4 cm h^{-1}). The optimized saturated hydraulic conductivity in the field is a profile representative hydraulic conductivity and is controlled by the most restricting layer. The small variation in saturated hydraulic conductivity between locations in the field suggests that the controlling layer is the same across the field.

Use of linear scaling in (1) precludes the determination of individual moisture content (θ) from reference θ^* in absence of known individual θ_r values. Equation (1) can, however, be used to determine $\theta - \theta_r$. Rather than compare the individual hydraulic functions obtained in the laboratory with those obtained from field reference profile functions and scaling factors, we present the mean and standard deviation of $(\theta - \theta_r)$ at selected soil water pressure head values in Table 3. The mean value of $(\theta - \theta_r)$ for the 0.6-m depth laboratory data is within 4 volume percent of the field data, while the 0.3-m depth mean laboratory value differ by as much as 10 volume percent from the in situ reference curves of methods 1 and 2. The variation in $(\theta - \theta_r)$ of the laboratory data has the same magnitude as that of the field data in the 0- to −100-cm soil water pressure head range, but

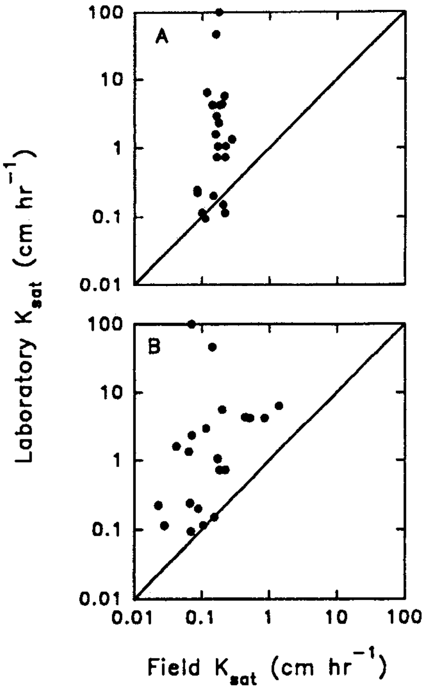

Figure 3. Comparison of estimated saturated hydraulic conductivity values from field data with laboratory data. (a) Field method 1. (b) Field method 2.

Table 2. Statistical Properties of Field and Laboratory (0.6 m) Saturated Hydraulic Conductivity Sand Group

Statistic	Laboratory	Field Method 1	Field Method 2
Mean, cm h^{-1}	8.46	1.70×10^{-1}	2.22×10^{-1}
Coefficient of variation	6.80	3.30×10^{-1}	1.35
Minimum, cm h^{-1}	9.40×10^{-2}	8.50×10^{-2}	1.80×10^{-2}
Maximum, cm h^{-1}	1.00×10^{2}	2.8×10^{-1}	1.40

Table 4. Statistical Properties of Scaling Factors From Simultaneous Scaling of Group 1

Statistic	Laboratory Data Combined	Laboratory Data 0.3 m	Laboratory Data 0.6 m	Field (Method 2)
Mean	1.00	1.00	1.00	1.00
Variance	1.70	1.40	0.87	0.23
Coefficient of variation	1.30	1.20	0.94	0.48

is greater for pressure head values smaller than −200 cm. Differences in variability between field profile and soil core data is a result of differences in observation scales. There is likely to be a larger variability in soil hydraulic properties between the relatively small soil cores as compared to the variability between entire soil profiles. As was noted earlier, variability in field properties decreases as soil properties are integrated over a larger sample. Statistical properties of the scaling factors obtained by simultaneous scaling of soil water retention and unsaturated conductivity functions are presented in Table 4. The data show that there is a large difference between the variances of the laboratory and field data. Application of the F test to these data indicated that the variance of the laboratory is statistically larger than the field data at the 99% confidence level.

Cumulative drainage has been used in this study to obtain scaling factors and reference profile hydraulic functions, but the scaling technique described in this study is applicable to other flow processes as well, for example, infiltration [*Vogel et al.*, 1991]. Reference profile infiltration rate obtained by scaling can be used as input for the estimation of reference profile soil hydraulic properties employing the inverse solution as was suggested by *Russo et al.* [1991]. The scaling factors obtained can then be used to estimate spatial distribution of soil hydraulic properties in a spatially variable field. Scaling factors obtained from infiltration most likely represent the soil hydraulic properties of the upper part of the soil profile rather than those of the whole profile. When infiltration is used, q and Q (equations (6) and (7)) now represent the infiltration rate and the cumulative infiltration, respectively.

Summary and Conclusions

The task of directly determining soil hydraulic functions at several locations within a large field is time consuming and tedious. We have shown how in situ soil hydraulic functions can be estimated quickly and with a minimum of number of soil data by combining the speed of the inverse solution technique with the linear variability concept. Within this methodology, it is no longer necessary to determine the hydraulic functions at every location in the field. One has only to carry out the inverse solution once to obtain the reference soil profile hydraulic functions, from which hydraulic functions at other locations can be estimated using scaling factors. The reference profile hydraulic functions can also be used as input to a numerical flow model where the scaling approach allows model input at locations other than the reference profile.

Results from the 0.6-m-depth laboratory analysis approximated the field data much better than those from the 0.3-cm-depth analysis. If soil hydraulic data of soil cores are used to reflect in situ soil behavior, the sampling depth should be considered. Infiltration may be represented sufficiently by soil cores from near the soil surface. As we have shown here, profile representative soil hydraulic functions may be represented better by cores obtained from deeper depths.

The variance of the estimated soil hydraulic function portrays the relationship between the laboratory and the field scale. The presented field method with integration over the 2.1-m soil profile reduces the spatial heterogeneity as measured in the small laboratory core samples. The new technique presented herein saves considerable amount of time, both computationally and in the field. Although cumulative drainage is used in this study to obtain the scaling factors and the reference profile hydraulic functions, the scaling technique employed can be used with other time-dependent functions such as field infiltration or laboratory outflow. The measurement of such dynamic variables is usually easier and faster than the direct measurement of hydraulic functions.

The application of the linear variability concept assumed identical initial pressure head profiles and equal pressure head values at the boundaries of the one-dimensional soil profiles. In the presented application, the initial pressure

Table 3. Means and Standard Deviations of $\theta - \theta_r$ Group 1

Soil Water Pressure, cm	Mean $\theta - \theta_r$, $m^3\ m^{-3}$: Laboratory Data 0.3 m	Laboratory Data 0.6 m	Field Method 1	Field Method 2	Standard Deviation $\theta - \theta_r$, $m^3\ m^{-3}$: Laboratory Data 0.3 m	Laboratory Data 0.6 m	Field Method 1	Field Method 2
0.0	0.275	0.316	0.367	0.338	0.069	0.069	0.079	0.066
−50.0	0.214	0.210	0.254	0.231	0.062	0.077	0.055	0.066
−100.0	0.171	0.125	0.114	0.083	0.071	0.084	0.077	0.056
−200.0	0.131	0.070	0.038	0.025	0.076	0.076	0.010	0.030
−300.0	0.112	0.051	0.018	0.014	0.075	0.067	0.003	0.020

distribution after irrigation and pressure head at the lower boundary during the whole measurement period were sufficiently close to zero for the measurement locations to satisfy these assumptions. The excellent agreement between the two field methods corroborates this. Hence in the presence of a shallow water table the linear variability concept can be successfully applied to draining soil profiles. Moreover, simultaneous application of the inverse solution to the draining reference soil profile reduces the number of required optimizations to one.

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