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## A COMPARISON OF VARIOUS METHODS TO SCALE SOIL HYDRAULIC PROPERTIES\*

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### ABSTRACT

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Soil water characteristic curves and hydraulic conductivity functions at 57 locations were scaled by various methods. Optimum scaling results were obtained when pressure head and conductivity data were scaled by different methods. Criteria that supported this conclusion were: (1) the percent reduction in sum of squares; (2) the correlation between unscaled and estimated pressure head values as obtained from the scaled mean hydraulic functions; and (3) correlation between scaling factors determined from soil water characteristic data and those determined from hydraulic conductivity data. Both sets of scaling factors were found to be lognormally distributed. A correlation of  $R^2 = 0.761$  was obtained between scaling factors determined from soil water characteristic data and those determined from hydraulic conductivity data. For the optimum scaling methods, the sum of squares, about the average curve, was reduced by 73 and 42%, for the water characteristic and hydraulic conductivity data, respectively. The results suggest that scaling can successfully be used for describing the variability of soil hydraulic properties of different soil map units and horizons.

### INTRODUCTION

Studies in soil water movement are increasingly concerned with the spatial variation of soil physical properties. The effects of spatially variable soil water characteristics and hydraulic conductivity functions were examined by Warrick et al. (1977a) and Warrick and Amoozegar-Fard (1979). The consequences of soil heterogeneity on water budget components in a small watershed were investigated by Peck et al. (1977) and verified by Clapp et al. (1983).

The results of this study will be used to determine the influence of spatially variable soil hydraulic properties on actual evapotranspiration rates in a watershed in The Netherlands. In dry years, a major contribution of the plant water uptake is supplied by upward water flow from a water table.

In many studies, spatial variation of the soil hydraulic properties is

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expressed by scaling factors. The definition of scaling factors comes from the work of Miller and Miller (1955, 1956). They introduced the similar media concept which is based upon assumptions concerning the microscopic geometric structure of porous media. Similar media differ only in the scale of their internal microscopic geometries and have therefore equal porosities. In principle, the similar media concept allows results, either experimental or computed, of soil water behaviour in one soil to be used to describe the behaviour in another by employing reduced variables defined in terms of appropriate microscopic characteristic lengths.

The purpose of scaling is to simplify the description of statistical variation of soil hydraulic properties. By this simplification, the pattern of spatial variability is described by a set of scale factors  $\alpha_r$ , relating the soil hydraulic properties at each location  $r$  to a representative mean. Youngs and Price (1981) extended the similarity concept to geometrically dissimilar soils. However, in that case scale factors obtained from  $h$  data are likely to be different from those obtained from  $K$  data.

Methods to determine scale factors were described by Warrick et al. (1977b) and Russo and Bresler (1980). These methods are based on regression analysis and can also be described as functional normalization methods (Tillotson and Nielsen, 1984). Peck et al. (1977), Lascano and Van Bavel (1982), and Ahuja et al. (1984) have shown how the distribution function of scale factors can be used to assess the effects of variable soil hydraulic properties on soil water flow.

The objective of this study was to compare several scaling methods as used to obtain average hydraulic functions and distributions of scale factors from experimental data.

## THEORY

Peck et al. (1977) defined a scaling parameter  $\alpha_r$  as the ratio of the microscopic characteristic length  $\lambda_r$  of a soil at location  $r$  and the characteristic length  $\lambda_m$  of a reference soil, or:

$$\alpha_r = \lambda_r / \lambda_m \quad (1)$$

where  $r = 1, \dots, R$  denote locations. As a result of the scaling theory one can relate the soil water characteristic and hydraulic conductivity function at given water contents at any location  $r$  to a mean  $h_m$  and  $K_m$ , such that for the soil water pressure head (negative in the unsaturated zone):

$$h_r = h_m / \alpha_r \quad (2)$$

and for the hydraulic conductivity:

$$K_r = K_m \alpha_r^2 \quad (3)$$

For similar media, eqns. (2) and (3) are true for all  $h_r$  and  $K_r$  measured at different water contents. Owing to the fact that soils do not have identical values of porosity,  $h$  and  $K$  are written as a function of degree of saturation  $S$

rather than volumetric water content  $\theta$ . The examined scaling methods differ in how  $h_m(S)$  and  $K_m(S)$  are determined. In this paper the factors  $\alpha_r$  for a given soil  $r$  will be treated as constants that are to be derived from observations of  $h_r(S)$  and  $K_r(S)$ , whereas the reference  $\alpha_m$  represents the mean curves for these functions.

### Method I

The first method is identical to the procedure of scaling  $K$ , which was described by Warrick et al. (1977b, page 358). When taking logarithms, eqns. (2) and (3) result in:

$$\log h_m(S_i) = \log h_r(S_i) + \log \alpha_r \quad (4)$$

$$\log K_m(S_i) = \log K_r(S_i) - 2 \log \alpha_r \quad (5)$$

where both  $h$  and  $K$  are expressed as a function of degree of saturation  $S$ , and  $i = 1, \dots, I(r)$  denote the different pressure steps at location  $r$ .  $\log h_m$  and  $\log K_m$  were estimated as a function of  $S$  by a third degree polynomial, thereby using the  $h(S)$  and  $K(S)$  data of all  $R$  locations:

$$\log h_m = a_0 + a_1 S + a_2 S^2 + a_3 S^3 \quad (6)$$

$$\log K_m = b_0 + b_1 S + b_2 S^2 + b_3 S^3 \quad (7)$$

Minimization of:

$$\sum_{r,i} [\log h_m(S_i) - \log \alpha_r - \log h_r(S_i)]^2 \quad (8)$$

and:

$$\sum_{r,i} [\log K_m(S_i) + 2 \log \alpha_r - \log K_r(S_i)]^2 \quad (9)$$

leads to a mean or reference soil water characteristic curve and hydraulic conductivity function and two sets of scale factor values, which are each normalized such that for these mean curves:

$$\alpha_m = 1/R \sum_{r=1}^R \alpha_r = 1 \quad (10)$$

### Method II

The second method is derived from Peck et al. (1977) who showed that, provided  $\alpha_r$  has an arithmetic mean 1:

$$h_m(S_i) = R \left\{ \sum_{r=1}^R [1/h_r(S_i)] \right\}^{-1} \quad (11)$$

and:

$$K_m(S_i) = (1/R^2) \left[ \sum_{r=1}^R K_r(S_i)^{1/2} \right]^2 \quad (12)$$

Given eqns. (11) and (12), scale factors can be calculated from:

$$\alpha_r^h = 1/I(r) \sum_{i=1}^{I(r)} h_m(S_i)/h_r(S_i) \quad (13)$$

$$\alpha_r^K = \left[ 1/I(r) \sum_{i=1}^{I(r)} K_r(S_i)/K_m(S_i) \right]^{1/2} \quad (14)$$

Hence,  $\alpha_r^h$  and  $\alpha_r^K$  are averaged over the measurements  $I(r)$ .

### Method III

In the third method, the mean hydraulic functions were found by fitting Van Genuchten's (1980) analytical expression for the soil water characteristic curve through the water retention data of all  $R$  locations combined. The mean hydraulic conductivity function can be predicted from the fitted water retention curve according to:

$$\Theta = \frac{\theta - \theta_{\text{res}}}{\theta_s - \theta_{\text{res}}} = [1 + |\beta h|^n]^{-m} \quad (15a)$$

$$m = 1 - (1/n) \quad (15b)$$

and:

$$K_r = K/K_s = \Theta^{1/2} [1 - (1 - \Theta^{1/m})^m]^2 \quad (16)$$

where  $\theta_{\text{res}}$ ,  $\theta_s$  and  $K_s$  denote the residual and saturated water content, and saturated hydraulic conductivity, respectively, and where  $\beta$ ,  $n$  and  $m$  are empirical parameters estimated by the model. Scale factors were determined from eqns. (13) and (14), but could also be obtained from minimization of the sum of squares as was done by Ten Berge [1986, eqns. (4)–(11)] to scale water retention data. The arithmetic means of these sets of scale factors, however, are not necessarily equal to 1.

### Method IV

The fourth method to scale soil water retention data only, was introduced by Warrick et al. (1977b). They minimized the sum of squares following from eqn. (2):

$$SS = \sum_{r,i} [h_m(S_i) - \alpha_r h_r(S_i)]^2 \quad (17)$$

by differentiating eqn. (17) with respect to each  $\alpha_r$ , setting each of the results equal to zero, and solving the linear system, subject to the constraint defined

by eqn. (10). In this study the mean values  $h_m(S_i)$  were estimated using eqn. (6), instead of the expression employed by Warrick et al. [1977b, eqn. (11)].

### *Method V*

Finally, method IV was also used to scale water content instead of pressure head values. There is no sound physical basis for doing this, however, it provides another means to express the variability of a set of soil water characteristic curves. While scaling  $\theta$ , it is assumed that water content values of the soil water characteristic curve can be scaled by:

$$\theta_r = \theta_m / \gamma_r \quad (18)$$

Equation (17) then transforms to:

$$SS = \sum_{r,i} [\theta_m(h_i) - \gamma_r \theta_r(h_i)]^2 \quad (19)$$

where  $\gamma_r$  denotes a scaling parameter for  $\theta$ . Such an approach was followed by Rao et al. (1983).

## MATERIALS AND METHODS

The scaling methods are compared for experimental  $\theta(h)$  and  $K(\theta)$  data, representing the soil physical properties of the Hupselse Beek watershed. This watershed is situated in the eastern part of The Netherlands and has been an experimental study area for almost 20 years. The Hupselse Beek watershed covers 650 ha of which 70% pasture and 20% arable land. The slightly loamy sands within the watershed area have groundwater tables within the first few meters below the soil surface. Throughout the whole region Miocene clay and low permeable glacial till layers are found beneath the sandy deposits at depths varying between 0.2 and a few meters below the soil surface.

Soil hydraulic properties were determined for various horizons at three different scales of observation. In the first sampling scheme, seven profiles across the 650 ha study area were sampled. These seven sites were chosen such that they included most of the characteristic soil profiles and horizons in the watershed. The results of the soil physical measurements, as well as the soil survey of the watershed were reported by Wösten et al. (1983). The second sampling scheme comprised an area of 0.5 ha and was chosen such that the seven sampled sites within this area were all from the same soil map unit. Samples were taken in duplicate in two horizons. Finally, the highest sampling density was achieved in sampling scheme 3, where six sites were sampled in triplicate within 2 m<sup>2</sup>, and in three horizons. This sampled area was located in the second scheme.

Soil water characteristics were obtained in the laboratory by the suction method and in-situ by simultaneous measurement of soil water tension and volumetric water content by tensiometers and neutron probe, respectively.

Hydraulic conductivity functions were determined by the crust test (Bouma, 1977), the hot-air method (Van Grinsven et al., 1985) and/or the sorptivity method (Dirksen, 1979). Before scaling, the soil hydraulic properties of the replicates were combined and fitted by eqns. (15) and (16) of the Van Genuchten model (1980). In total, the soil water characteristic and hydraulic conductivity curves of 57 locations were scaled. Details of measurement schemes and techniques can be found in Hopmans and Stricker (1987).

## RESULTS AND DISCUSSION

The degree of success of each scaling method can be deduced from a comparison of the percent reduction in sum of squares  $SS$  of deviations between the scaled mean hydraulic function and the individual hydraulic data for each location sampled, before and after scaling. If scaling is successful, the resulting  $SS$  value should be lower than for the unscaled hydraulic data. However, the comparison of reduction in  $SS$  values is only of limited use, since its value will depend on the definition or form of  $h_m$  and  $K_m$ . Table 1 which lists these reductions in  $SS$  values, makes distinction between "measured" and "fitted" water retention and conductivity data. The latter were obtained by using eqns. (15) and (16) of the Van Genuchten model (1980).

At first glance, it can be seen that the use of fitted rather than the original measured hydraulic data increases the percent reduction in  $SS$ . This result was expected, since the fitted hydraulic characteristics for most locations were obtained from a combination of two or three replicates. The higher scattering of measured  $\theta(h)$  and  $K(\theta)$  data as compared to the fitted hydraulic data will reduce the effectiveness of scaling and therefore decreases the percent reduction in  $SS$ .

TABLE 1

Comparison of scaling methods

Method	Reduction in $SS$ (%)	$R^2$ value unscaled versus estimated $h$ or $K$
I $h$ fitted	54	0.850
$K$ fitted	42	0.555
$K$ measured	35	0.341
*II $h$ fitted	97	0.346
$K$ fitted	28	0.168
*III $h$ fitted	98	0.415
$K$ fitted	no reduction	0.279
IV $h$ fitted	73	0.966
$h$ measured	62	0.832
V $\theta$ fitted	72	0.888

\* Soil hydraulic properties were scaled for  $S$  values between 0.4 and 1.0.

Striking are also the differences between percent reduction values of methods II and III as compared with the other methods. Table 1 would indicate that the scaled water retention data by methods II and III are superior. However, methods II and III differ conceptually from the other methods, as will be explained hereafter.

Scale factor values in methods II and III were calculated for a range of degree of saturation  $S$ , equal for all locations [eqn. (13)]. Since soil water retention data were highly variable (Fig. 1a), the use of these scaling methods implicated that pressure head values at low  $S$  were needed from extrapolation for soils, with available pressure head data at only high  $S$  values. This extrapolation (Van Genuchten model) would generally lead to large absolute values of  $h$  and, therefore, to a disproportional reduction in  $SS$  as compared with methods I and IV. The failure of method III to scale  $K$  data is attributed to a bad prediction of the mean conductivity function by the Van Genuchten model.

Table 1 and the analysis so far would indicate that methods IV and I are to be preferred to scale water retention and conductivity data, respectively. Also method V yields a high reduction in  $SS$ . However, there is no sound soil physical basis that justifies the scaling of  $\theta$  rather than  $h$ . The latter method can, however, be used to obtain an average soil water characteristic curve.

Another criterion that might be useful in comparing various scaling methods is the value of the correlation coefficient when unscaled pressure head and conductivity values are regressed versus pressure head and conductivity values estimated from the scaled mean soil water characteristic curve and hydraulic conductivity function, using eqn. (2) or (3). A high correlation coefficient would indicate that the scaling method has not distorted the hydraulic data as much as compared with a scaling method with a low  $R^2$  value. Also these  $R^2$  values are shown in Table 1 (third column). Again, methods IV ( $h$  data) and I ( $K$  data) were superior to the other methods.  $R^2$  values for methods II and III were very low. The unscaled and scaled soil water characteristic and conductivity data (methods IV and I, respectively) are shown in Figs. 1 and 2. From Fig. 2 it is obvious that the variation in  $K_s$  did not decrease after scaling. The large variation in  $K_s$  values between locations is not consistent with the much lower variability of scaled unsaturated conductivity values. This is most likely due to the larger within location variability of  $K_s$ .

If the soils at the sampled locations of the watershed were similar media, then the set of  $\alpha_r$  calculated from  $h$  data ( $\alpha_r^h$ ) should be identical to those calculated from  $K$  data ( $\alpha_r^K$ ). Hence, a plot of  $\alpha_r^h$  versus  $\alpha_r^K$  would indicate which scaling method approaches the conditions set by eqns. (2) and (3) the best. Beforehand, it was known from a soil survey, that the sampled locations were classified as different soil map units. So, it was not expected that a plot of the two sets of scaling factors would fall exactly along the 1:1 line. Values of  $\alpha_r^h$  for all 57 locations determined with method IV are compared in Fig. 3 with  $\alpha_r^K$  determined with method I. A correlation coefficient  $R^2$  of 0.761 has been calculated. The slope of the line, emanating from the origin was calculated to be 0.898.  $R^2$  values and slopes for the regression lines of methods I, II and III and

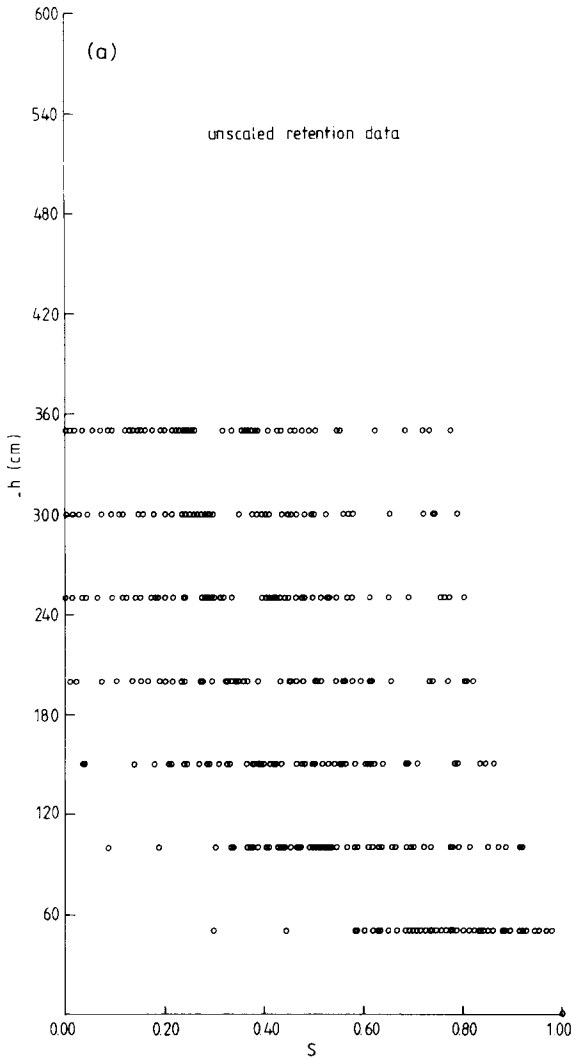


Fig. 1. Unscaled (a) and scaled (b) water retention data with method IV.

the combination of I-IV are listed in Table 2. A value of the slope nearest to 1 was found for method I.

Scale factors as obtained from methods I and IV are compared with those obtained with method II and plotted in Fig. 4. If both methods would yield the same scaling results then the two sets of scaling factors would fall on the 1:1 line. The relationship between scale factors as determined by methods IV and V is strongly nonlinear (Fig. 5). The range of  $\alpha_r^\theta$  values obtained when scaling  $\theta$  instead of  $h$  appears to be much smaller. Similar results were reported by Rao



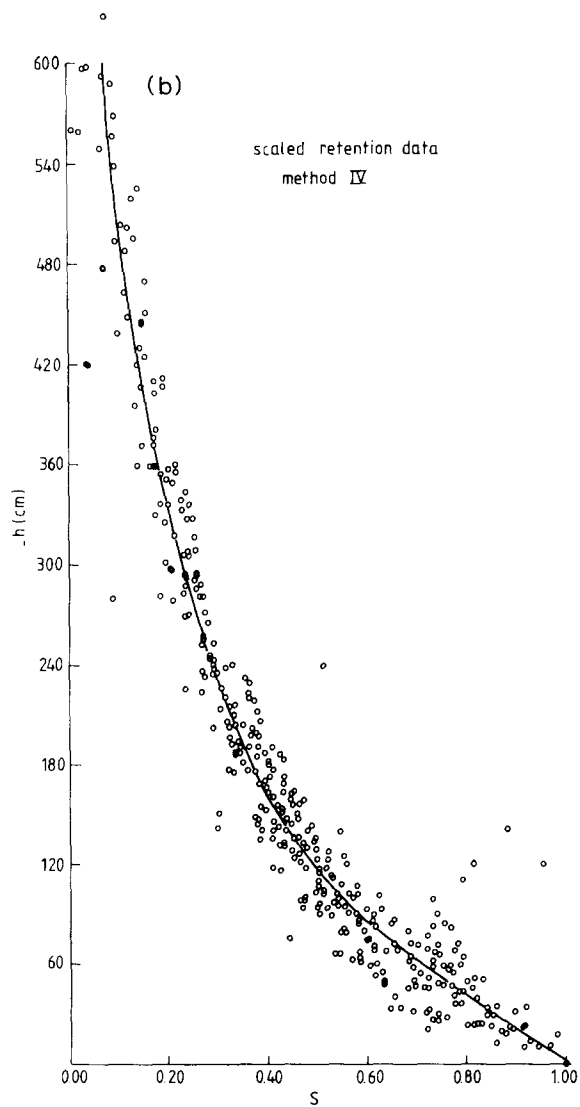


TABLE 2

$R^2$  values and slopes (intercept = 0) for regression of  $\alpha_r^h$  versus  $\alpha_r^k$

Method	$R^2$	Slope
I	0.760	0.989
II	0.765	1.078
III	0.756	1.298
I-IV fitted	0.761	0.898
I-IV measured	0.587	0.858

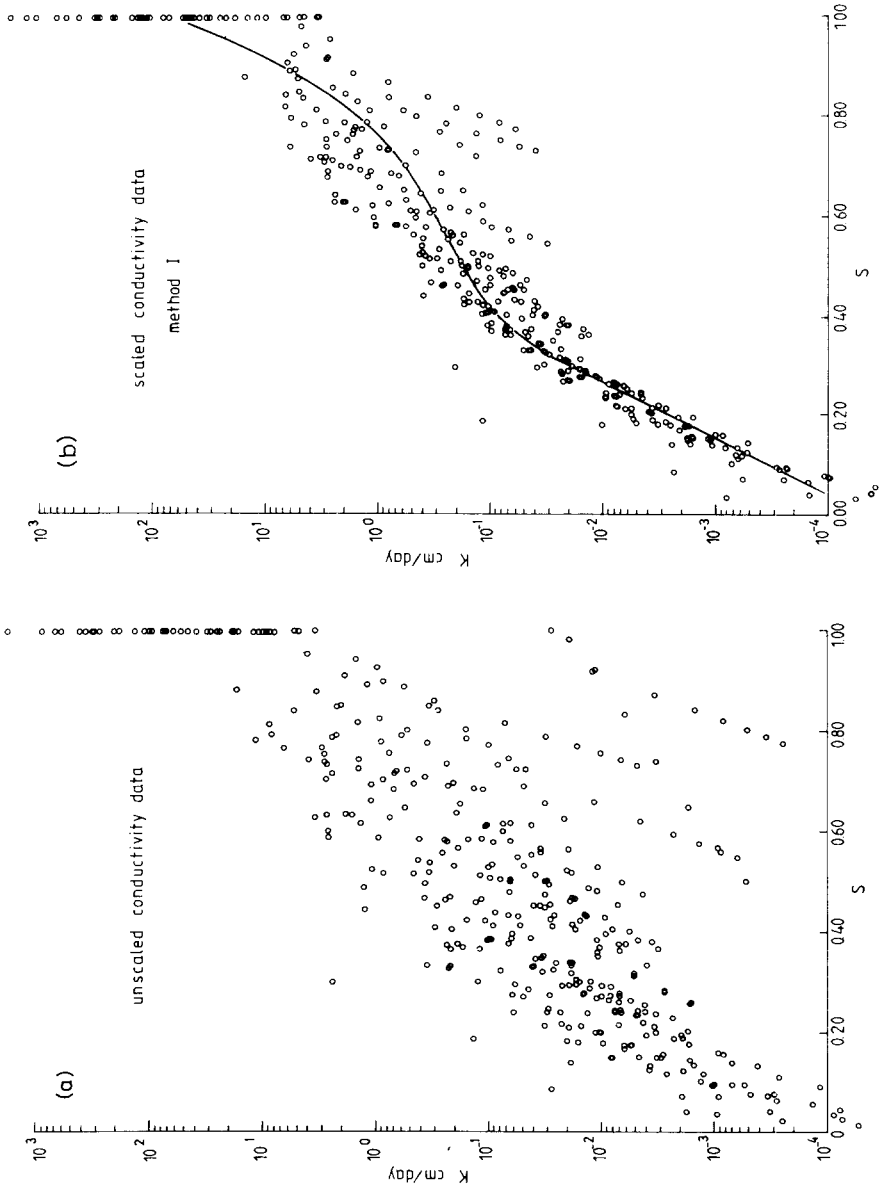


Fig. 2. Unscaled (a) and scaled (b) conductivity data with method I.

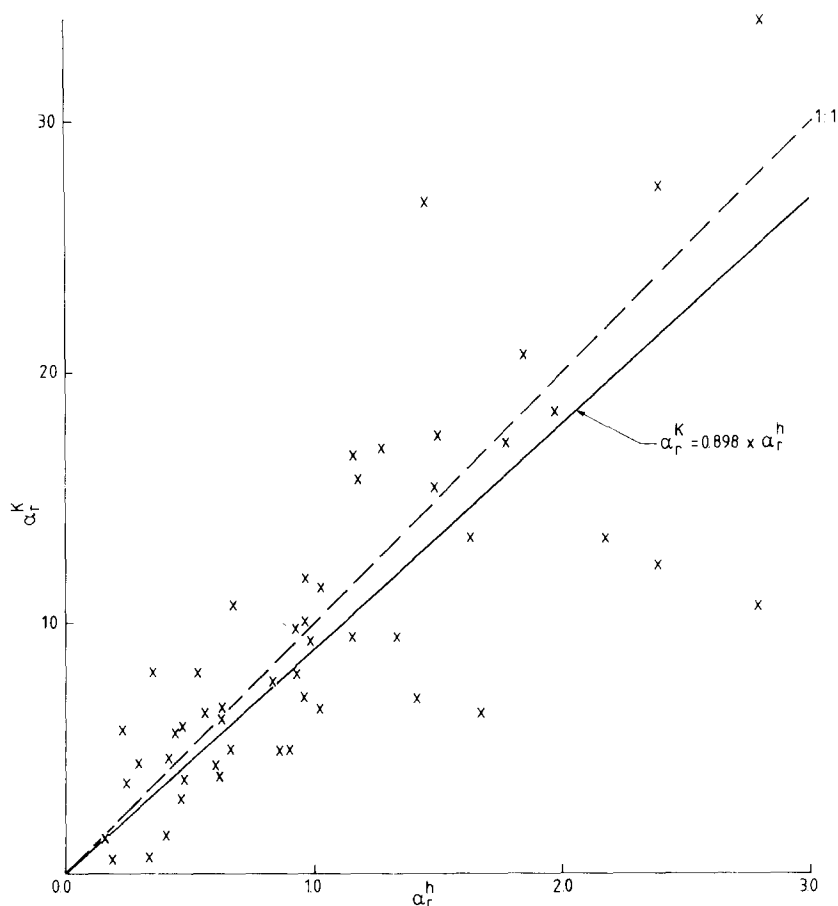


Fig. 3. Values of  $\alpha_r$ , calculated from hydraulic conductivity data (method I) versus those calculated from soil water characteristic data (method IV).

et al. (1983). Hence, one expects the distribution of  $\alpha_r$  values as obtained with method V to be different from the other methods.

Table 3 compares the statistics (estimated mean  $\mu$  and standard deviation  $\sigma$ ) for the normal and lognormal distribution of the set of scale factor values determined by the five scaling methods. In this table, *KS* denotes the modified Kolmogorov–Smirnov statistic (Stephens, 1974) which is used to test whether the empirical distribution function and the considered hypothetical distribution function are significantly different at a specified confidence level. The empirical distribution function of  $\alpha_r$  was accepted as being not different from the hypothetical distribution (normal or lognormal), if the *KS* value was smaller than 0.819 (at 90% confidence level).

Clearly we would like the distributions of  $\alpha_r^h$  and  $\alpha_r^K$  to be the same. Table 3 suggests that  $\alpha_r^h$  and  $\alpha_r^K$  have different distributions functions when the same scaling method is used for both  $h$  and  $K$  (Methods I, II and III). Rao et al. (1983)

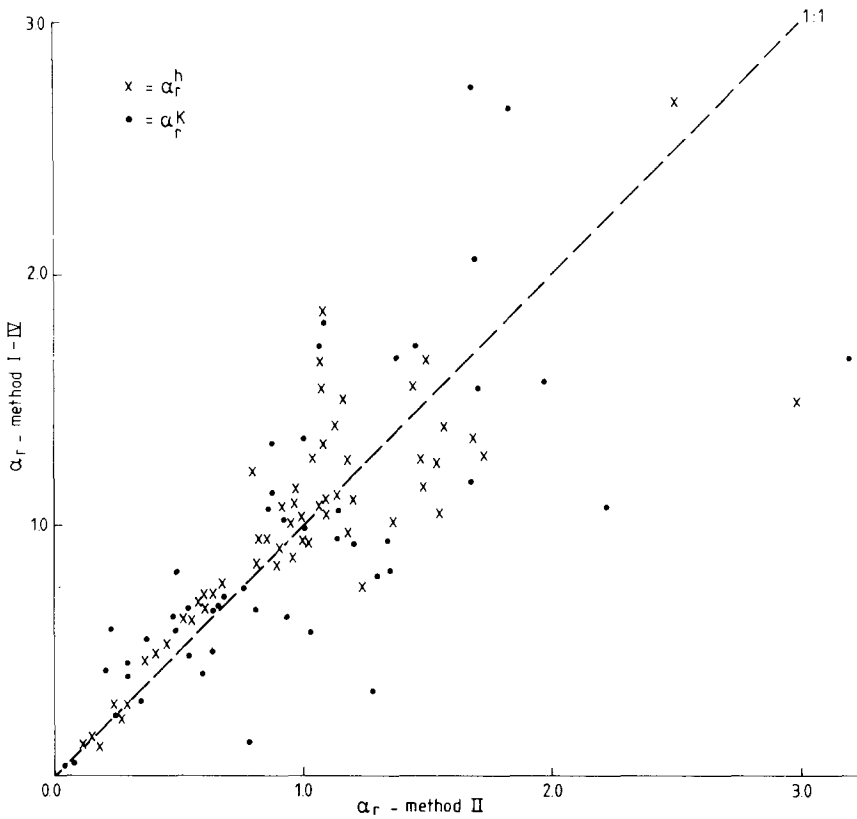


Fig. 4. Comparison of scaling methods II and I-IV for soil water characteristic and conductivity data.

used methods I and V to scale conductivity and retention data, respectively. Table 3 shows that the distribution functions of  $\alpha$  for these two methods are different, and can, therefore, not be used to test the similarity concept. Again, the combination of method I (to scale  $K$  data) and method IV (to scale  $h$  data) seems to be the most promising technique. Each method results in a low  $KS$  value of 0.647 and 0.576 for the lognormal distribution functions of  $\alpha_r^K$  and  $\alpha_r^h$ , respectively. Fractile diagrams of the  $\alpha_r$  and  $\log(\alpha_r)$  values, corresponding to the two methods are shown in Fig. 6. The fact that the plot of  $\log(\alpha_r)$  is closer to the straight line indicates that a lognormal distribution fits the results more adequately than a normal distribution. The coefficients of variation of  $\alpha_r$ , based on the lognormal distributions were 0.574 and 0.466 for the  $K$  and  $h$  data, respectively. Those values are somewhat larger than assumed by Peck et al. (1977) and Russo and Bresler (1980), but smaller than reported by Warrick et al. (1977b).

When analyzing the distribution function of scale factor values for all soils, it was assumed that the scale factors are statistically homogeneous and uncorrelated variables, independent of their spatial position. A more complete approach should take into account the correlation which may exist between

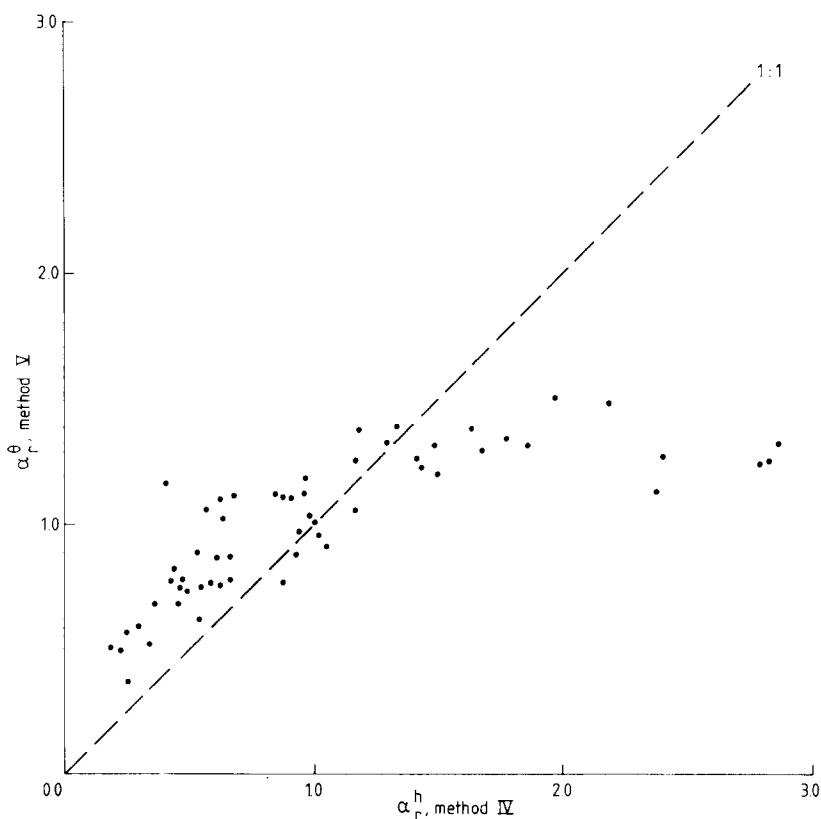


Fig. 5. Values of  $\alpha_r$  from soil water characteristic data, as calculated with method V versus those calculated with method IV.

TABLE 3

Comparison of distribution of  $\alpha_r$  values for the five scaling methods

Method		Normal			Lognormal		
		$\mu$	$\sigma$	KS	$\mu$	$\sigma$	KS <sup>a</sup>
I	$h$ fitted	1.0	0.4561	0.591 <sup>b</sup>	-0.0605	0.2625	1.380
	$K$ fitted	1.0	0.7002	1.045	-0.1131	0.3518	0.647 <sup>b</sup>
	$K$ measured	1.0	0.7899	1.031	-0.1267	0.3608	0.562 <sup>b</sup>
II	$h$ fitted	1.0	0.5337	0.918	-0.0741	0.2852	1.448
	$K$ fitted	1.0	0.9012	1.131	-0.1088	0.3822	0.811 <sup>b</sup>
III	$h$ fitted	0.9950	0.5408	1.00	-0.0805	0.2958	1.479
	$K$ fitted	1.230	1.192	1.493	-0.0652	0.4116	0.718 <sup>b</sup>
IV	$h$ fitted	1.0	0.6550	1.145	-0.0915	0.2922	0.576 <sup>b</sup>
	$h$ measured	1.0	0.6749	1.070	-0.0847	0.2717	0.659 <sup>b</sup>
V	$\theta$ fitted	1.0	0.2768	0.693 <sup>b</sup>	-0.0192	0.1362	0.940

<sup>a</sup>Critical region:  $KS > 0.819$  at 90% confidence level.

<sup>b</sup>Acceptable fit.

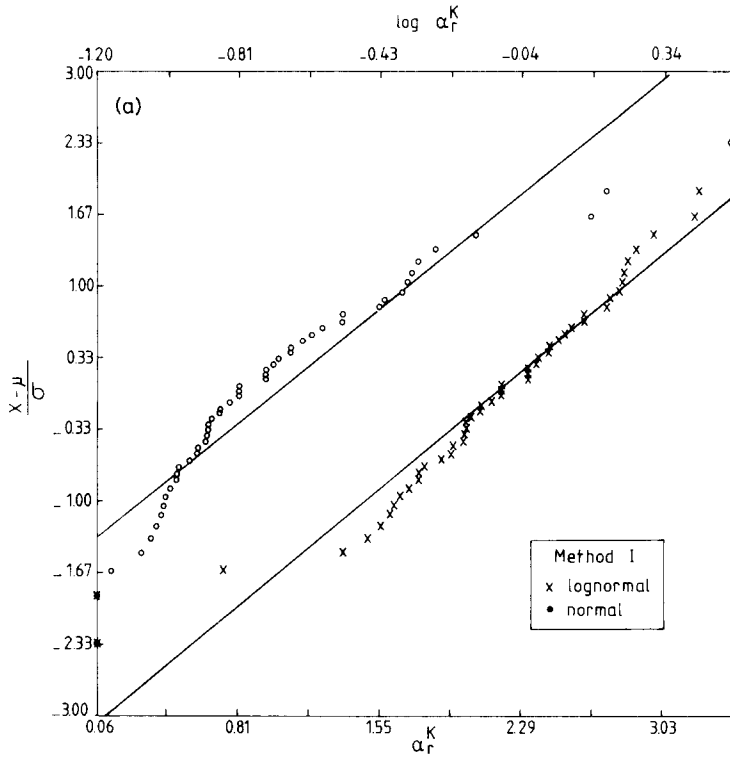


Fig. 6. Fractile diagrams of  $\alpha_r$  and  $\log \alpha_r$  calculated from conductivity data (a) and soil water characteristic data (b).

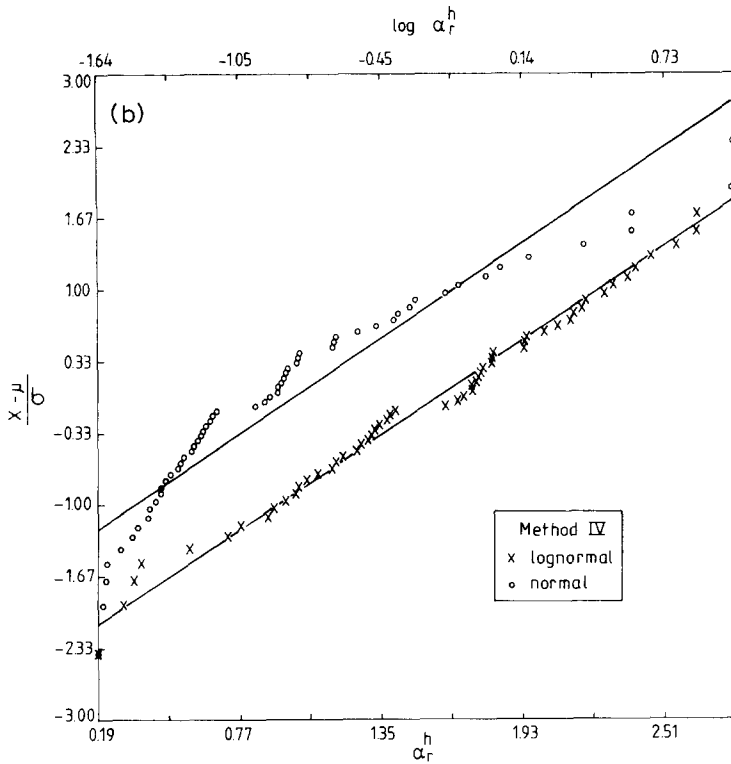
nearby measurements, before analyzing the distribution of the scale factor values. After differentiation between horizons, it was later found that the distance between locations over which scale factor values are spatially dependent is of the order of 10 m.

CONCLUSIONS

The results and comparisons of various scaling methods suggest that scaling of  $h$  and  $K$  data should be done by two different methods. Both methods, which were introduced by Warrick et al. (1977b), yield a lognormal distribution of scale factors.

Although the sampled locations were located in different soil map units and horizons, scaling was successful with regard to the possibility of representing the variability of soil hydraulic properties by a set of scale factor values.

The scaled mean soil-water characteristic curve and hydraulic conductivity function can be viewed as being the representative means of the scaled hydraulic data. From the estimated distribution of scale factor values one may generate a new set of scale factors, representing the variability of the soil hydraulic properties of the sampled area.



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