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Porous Media With Linearly Variable Hydraulic Properties

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The similar media concept is reformulated and extended to allow for analysis of unsaturated or saturated flow in a system of parallel nonhomogeneous one-, two-, or three-dimensional soil profiles. The resulting invariance of Richards' equation for a set of soil profiles is valid for certain initial and boundary conditions only. Simple techniques are outlined and demonstrated by examples for direct identification of scaling factors based on measured soil hydraulic data, and indirect identification based on measured dynamic characteristics of a soil water system. Suggested applications of the discussed concept are (1) the classification of soils according to the shape of their reference hydraulic properties and (2) the description of time and space variability of soil water properties for numerical modeling purposes.

INTRODUCTION

The use of the linear variability concept in soil physics has its roots in the work of *Miller and Miller* [1956] and was derived from the theory of similarity, which is well known in classical hydraulics. The computational techniques related to this concept, and often the concept itself, are referred to as scaling. The original concept was modified several times, the notion of similarity being gradually broadened. Similar media as introduced by Miller and Miller differ only in the scale of their internal geometry, having identical porosities. *Warrick et al.* [1977] extended the use of similar soils to soils with different internal geometries. The assumption of identical porosities was eliminated by using degree of saturation instead of volumetric moisture content. A new approach was introduced by *Simmons et al.* [1979]. They based their work on similarity between soil hydraulic functions, instead of similarity between respective internal geometries. A comprehensive discussion of the different approaches to scaling is given by *Tillotson and Nielsen* [1984]. The use of scaling for soil water modeling is reported, for example, by *Peck et al.* [1977], *Warrick and Amoozegar* [1979], *Sharma et al.* [1980], *Youngs and Price* [1981], and *Hopmans and Stricker* [1989].

The present paper is based on the functional similarity concept introduced by *Simmons et al.* [1979]. The scaling relations are generalized and extended to enable their incorporation in numerical modeling of water movement in systems of parallel stratified one-, two-, or three-dimensional soil profiles.

The space and time variability of soil hydraulic properties is considered to have two components, a linear and nonlinear component. In this paper we examine a fictitious porous medium with purely linear variability of hydraulic properties, expressed in terms of scaling factors. This variability is interpreted as an approximation of the linear component of real soil variability. The scaling factors cause a linear transformation on each of the involved variables. The unexplained variability after this transformation is assumed to be caused by the nonlinear component of the total variability. For example, in *van Genuchten*'s [1980] hydraulic relationships the nonlinear component is characterized by the parameter n and its variability. We assume that the linear component is dominating the nonlinear component. The concept allows the study of nonhomogeneous soil water systems in general. Furthermore, the linear variability concept yields relationships between variability of soil hydraulic properties and parameters that describe the variability of dynamic soil water flow processes as infiltration, redistribution and drainage.

LINEAR VARIABILITY CONCEPT

A soil is referred to as a linearly nonhomogeneous soil if its hydraulic properties obey the following rules:

1. The space and time variability of soil hydraulic properties can be expressed in terms of a linear transformation:

$$\begin{aligned} \mathbf{K}(T, \mathbf{r}, h) &= \alpha_K(T, \mathbf{r})\mathbf{K}^*(h^*) \\ \theta(T, \mathbf{r}, h) &= \theta_r(T, \mathbf{r}) + \alpha_\theta(T, \mathbf{r})[\theta^*(h^*) - \theta_r^*] \end{aligned} \qquad (1)$$

where

$$h = \alpha_h(T, \mathbf{r})h^*$$

T is an index of time and allows for temporal changes in the hydraulic functions; $\mathbf{r} \equiv (x, y, z)$ is a position vector with z positive upward, and $\mathbf{K}(h)$ and $\theta(h)$ are soil hydraulic

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TABLE 1. Comparison of Scaling Factors

Author	h	K	θ
Miller and Miller [1956]	$\lambda h = \lambda^{\cdot} h^{\cdot}$	$K/\lambda^2 = K^{\cdot}/\lambda^{\cdot 2}$	$\theta = \theta^{\cdot}$
Warrick et al. [1977]	$\alpha = h^{\cdot}/h$	$\alpha = (K/K^{\cdot})^{1/2}$	$S = \theta/\theta_s$
Simmons et al. [1979]	$\alpha_1 = h^{\cdot}/h$	$\alpha_2 = (K/K^{\cdot})^{1/2}$	$S = \theta/\theta_s$
this paper	$\alpha_h = h/h^{\cdot}$	$\alpha_K = K/K^{\cdot}$	$\alpha_\theta = (\theta - \theta_r)/(\theta^{\cdot} - \theta_r^{\cdot})$

The dotted variables denote reference values; λ and α denote scaling factors.

characteristics at point $\mathbf{r}$, i.e., the hydraulic conductivity–pressure head and soil moisture–pressure head relations. $\mathbf{K}^{\cdot}(h^{\cdot})$ and $\theta^{\cdot}(h^{\cdot})$ are space and time invariant reference soil hydraulic characteristics; α_K, α_θ and α_h are scaling factors associated with soil hydraulic conductivity, moisture content and pressure head, respectively, and θ_r denotes residual moisture content. Table 1 shows a comparison of the scaling factors with those commonly used in the literature.

2. The overall, point-to-point, variability, defined by (1), is decomposed into two independent components: a local (within a profile) and a global (between profiles) component:

$$\alpha_K(T, \mathbf{R}, \mathbf{r}) = \gamma_K(T, \mathbf{R})\beta_K(\mathbf{r})$$
$$\alpha_\theta(T, \mathbf{R}, \mathbf{r}) = \gamma_\theta(T, \mathbf{R})\beta_\theta(\mathbf{r}) \quad (2)$$
$$\alpha_h(T, \mathbf{R}, \mathbf{r}) = \gamma_h(T, \mathbf{R})\beta_h(\mathbf{r})$$

where the index $\mathbf{R}$ allows for global variability (profile identifier); the vector $\mathbf{r}$ accounts for local variability, and describes the position within the local (profile) coordinate system, and γ and β denote global and local components of the respective scaling factors α.

We define a soil profile as a one-, two-, or three-dimensional soil region as shown in Figure 1. An example of the space distribution of scaling factors within and between two one-dimensional soil profiles is given in Figure 2. For the one-dimensional case, the local component β is a function of z only. The difference between profiles is determined by the global component γ.

Having a set of soil profiles with hydraulic characteristics that vary according to the outlined linear variability concept, it is possible to define a reference soil profile as a profile for which

$$\gamma_K = \gamma_\theta = \gamma_h = 1 \quad (3)$$

The soil hydraulic properties of this reference profile are then fully characterized by the pair of functions $\mathbf{K}^*(\mathbf{r}, h^*)$ and $\theta^*(\mathbf{r}, h^*)$ obtained by combining (1), (2) and (3):

$$\mathbf{K}^*(\mathbf{r}, h^*) = \beta_K(\mathbf{r})\mathbf{K}^{\cdot}(h^{\cdot})$$
$$\theta^*(\mathbf{r}, h^*) = \theta_r^*(\mathbf{r}) + \beta_\theta(\mathbf{r})[\theta^{\cdot}(h^{\cdot}) - \theta_r^{\cdot}] \quad (4)$$

where

$$h^* = \beta_h(\mathbf{r})h^{\cdot}$$

Water movement in any soil profile as well as in the reference profile can be described by Richards' equation:

$$\frac{\partial \theta}{\partial t} = \operatorname{div}\,[\mathbf{K}(\operatorname{grad} h + \operatorname{grad} z)] \quad (5)$$

Provided that certain initial and boundary conditions are satisfied (see (12)) and that the solution for the reference profile is available, the dynamic variables of the other profiles can be determined from

$$\mathbf{v}(T, t, \mathbf{R}, \mathbf{r}) = \gamma_K(T, \mathbf{R})\mathbf{v}^*(t^*, \mathbf{r}^*)$$
$$\theta(T, t, \mathbf{R}, \mathbf{r}) = \theta_r(T, \mathbf{R}, \mathbf{r}) + \gamma_\theta(T, \mathbf{R})[\theta^*(t^*, \mathbf{r}^*) - \theta_r^*(r^*)] \quad (6)$$
$$h(T, t, \mathbf{R}, \mathbf{r}) = \gamma_h(T, \mathbf{R})h^*(t^*, \mathbf{r}^*)$$

where

$$t^* = \frac{\gamma_K(T, \mathbf{R})}{\gamma_\theta(T, \mathbf{R})\gamma_h(T, \mathbf{R})}\,t; \qquad \mathbf{r}^* = \frac{1}{\gamma_h(T, \mathbf{R})}\,\mathbf{r} \quad (7)$$

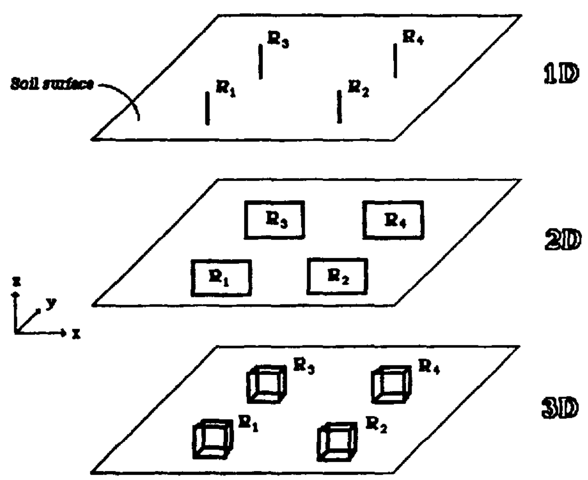

Fig. 1. Example sets of one-, two-, and three-dimensional soil profiles.

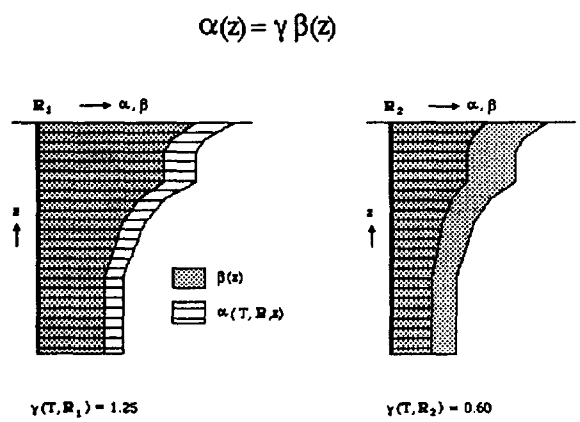

Fig. 2. Spatial distribution of an overall scaling factor α and its local and global components β and γ for two one-dimensional profile examples.

Relationships (6) can be proved by substituting (1), (2), (4), and (7) into (5). Since these relationships can be used to compute pressure head, Darcian flux, **v**, and moisture content at any point of any soil profile at time t from the distribution of respective reference variables at time t^*, they can also be regarded as a linear model to describe variability of the dynamic characteristics of a soil water system. The described procedure corresponds to the similar media theory derived by *Miller and Miller* [1956] for homogeneous soils, but uses three instead of one scaling factor and applies to nonhomogeneous soil profiles as well.

To avoid dependency between soil profile geometry (depth of soil profile, thickness of soil layers, etc.) and soil hydraulic properties through the presence of γ_h in the position vector transformation in (7), the following additional constraint is necessary:

$$\gamma_h(T, \mathbf{R}) = 1 \tag{8}$$

Consequently, the variability of the scaling factor α_h is restricted to its local component $\beta_h(\mathbf{r})$ only and $\mathbf{r}^* = \mathbf{r}$.

The question arises, if knowledge of the distribution of two scaling factors, γ_K and γ_θ, is sufficient to describe the global component of actual soil variability. The two parameters γ_θ and γ_K represent a relative measure of cross-sectional area available for water flow and permeability, respectively, and together contribute greatly to soil water flow.

A crucial point in the application of the described concept in soil water movement modeling is the formulation and interpretation of initial and boundary conditions imposed on a given system of soil profiles. The initial and boundary conditions for the reference profile can be written in conventional form. The initial condition for the reference profile is formulated by

$$h^* = \varphi^*(\mathbf{r}) \tag{9}$$

whereas pressure head and flux boundary conditions at any point of the boundary of the reference profile are defined by

$$h^* = \psi^*(t^*) \tag{10}$$

$$v^* = \rho^*(t^*) \tag{11}$$

respectively, where φ^*, ψ^* and ρ^* are prescribed functions of space and time. Initial and boundary conditions for a given system of soil profiles must satisfy the following conditions in order to conform to (6) and (8):

$$\begin{aligned} \varphi(T, \mathbf{R}, \mathbf{r}) &= \varphi^*(\mathbf{r}) \\ \psi(T, \mathbf{R}, t) &= \psi^*(t^*) \\ \rho(T, \mathbf{R}, t) &= \gamma_K(T, \mathbf{R})\rho^*(t^*) \end{aligned} \tag{12}$$

The first condition in (12) is easy to fulfill: The initial pressure head profile must be identical for all profiles of a given system but can vary locally. A similar statement applies for the second condition. No problems arise if we assume equal pressure head boundary values for corresponding boundary points of the different profiles. However, precautions are necessary if ψ is not constant in time. Pressure head changes at time t^* for the reference profile must then coincide with corresponding changes at time t of any other profile (see (7)). The most restrictive condition in (12) is the third one. In addition to the "event timing," as described for the pressure head boundary condition, the fluxes imposed on the different soil profiles must be linearly proportional (with ratio γ_K) to the corresponding flux imposed on the reference profile.

Although applications of the presented concept with respect to the modeling of soil water movement are restricted by the conditions in (12), some important applications are not affected. Examples are (1) constant pressure head infiltration into a stratified soil profile, (2) water redistribution within a soil water system with zero flux boundaries, and (3) drainage flow with constant suction head boundary condition (e.g., laboratory outflow experiment).

CALCULATION OF SCALING FACTORS

Scaling Factors for Soil Hydraulic Properties

Given a set of soil samples taken from different horizons of different soil profiles and given the soil hydraulic properties of these samples determined in the laboratory, overall (point-to-point) scaling factors (α), as defined by (1), as well as the hydraulic parameters of the reference soil can be identified by some of the commonly used scaling techniques that are based on least squares sum minimization (e.g., those reviewed by *Hopmans* [1987]). As an alternative, the following instructive and simple technique can be applied:

1. Retention and hydraulic conductivity data for each of the measured samples are independently fitted to analytical expressions. Alternatively, hydraulic conductivity curves are predicted from retention curves and saturated conductivity values by applying capillary-based models [e.g., *Mualem*, 1976].

2. Overall scaling factors are computed for each pair of soil hydraulic characteristics from the relationships

$$\begin{aligned} \alpha_K &= K_s/K_s^{\cdot} \\ \alpha_\theta &= (\theta_s - \theta_r)/(\theta_s^{\cdot} - \theta_r^{\cdot}) \\ \alpha_h &= h_c/h_c^{\cdot} \end{aligned} \tag{13}$$

where the reference saturated hydraulic conductivity $K_s^{\cdot}$, $(\theta_s^{\cdot} - \theta_r^{\cdot})$ and $h_c^{\cdot}$ are arithmetic means of the respective values of K_s, $(\theta_s - \theta_r)$ and h_c. The value of h_c is computed for each measured soil sample from

$$h_c = \frac{1}{(\theta_s - \theta_c)} \int_{\theta_c}^{\theta_s} -h(\theta)\, d\theta \tag{14}$$

where θ_c is quite arbitrarily chosen, but selected so that the majority of measured data points of all curves are in the interval (θ_c, θ_s). A value near the middle of the interval (θ_r, θ_s) is recommended for θ_c. In (14), h_c denotes the average h value within the interval (θ_c, θ_s).

3. The parameters of the reference soil hydraulic characteristics are then determined as follows. The coordinates $(\theta - \theta_r)$, h and K of each measured data point for each pair of soil hydraulic characteristics are divided by the respective scaling factors. The superposition of all transformed data points yields a single pair of reference soil hydraulic characteristics. Selected analytical expressions are fitted through both transformed data sets, and the resulting fitting parameters are referred to as the reference soil hydraulic parameters.

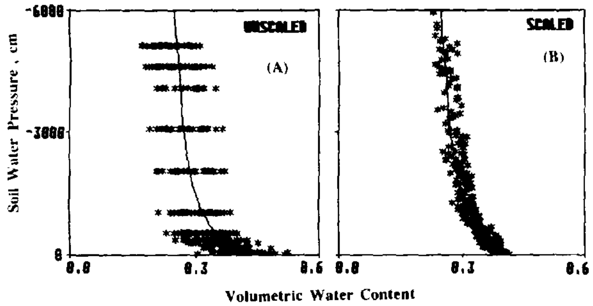

Fig. 3. Scaling of retention curves for Trebon soil. Symbols denote (*a*) unscaled and (*b*) scaled retention data; the solid line is the reference curve.

The following example illustrates the application of linear scaling to retention data. A set of 36 undisturbed 100-cm^3 soil cores was taken along a 500-m transect from an agricultural field in the Trebon region of southern Bohemia in Czechoslovakia. Retention data for the clayey loam soil were measured by pressure desorption. The linear scaling procedure was used to validate the linear variability assumption. Measured retention data were approximated by *van Genuchten*'s [1980] expressions. Retention data before and after scaling are shown in Figure 3. After the reference soil hydraulic parameters and individual scaling factors are determined, one can express the variability in terms of the reference characteristics and space-variable scaling factors using the transformations in (1).

Two different approaches can be used to decompose the point-to-point scaling factors into their local and global components. Firstly, one can study the deterministic distribution of scaling factors. Scaling factors can be grouped according to layer and soil profile. After averaging, one can compute the within-group and between-group scaling factors and interpret them as the local and global components. Secondly, one may study the stochastic distribution of scaling factors, and decompose their variability into local and global components by using statistical and geostatistical techniques, as by analysis of variance and semivariogram analysis.

Scaling Factors for Infiltration Curves

Equation (6) provides us with a means of investigating relationships between variability of soil hydraulic properties and variability of infiltration characteristics and of using those relationships to infer one from the other. From (6), (7) and (8) it follows that

$$v(T, t, \mathbf{R}, \mathbf{r}) = \gamma_K(T, \mathbf{R}) v^*(t^*, \mathbf{r}) \tag{15}$$

$$I(T, t, \mathbf{R}, \mathbf{r}) = \gamma_\theta(T, \mathbf{R}) \int_0^{t^*} v^*(t^*, \mathbf{r})\, dt^* = \gamma_\theta(T, \mathbf{R}) I^*(t^*, \mathbf{r}) \tag{16}$$

$$t = \frac{\gamma_\theta}{\gamma_K} t^* \tag{17}$$

where v and v^* now denote the infiltration rates at a boundary point of a selected soil profile and the reference profile, respectively and I denotes cumulative infiltration at that point.

Analogously to soil hydraulic properties, the variability within a set of cumulative infiltration curves for the same set of soil profiles can be described by a linear transformation:

$$\begin{aligned} v(T, t, \mathbf{R}, \mathbf{r}) &= \gamma_v(T, \mathbf{R}) v^*(t^*, \mathbf{r}) \\ I(T, t, \mathbf{R}, \mathbf{r}) &= \gamma_I(T, \mathbf{R}) I^*(t^*, \mathbf{r}) \\ t &= \gamma_t(T, \mathbf{R}) t^* \end{aligned} \tag{18}$$

Comparison of (15), (16), (17) with (18) yields

$$\gamma_v = \gamma_K \qquad \gamma_I = \gamma_\theta \qquad \gamma_t = \gamma_\theta/\gamma_K \tag{19}$$

The mutual relation between infiltration scaling factors can be determined from (19):

$$\gamma_v = \gamma_I/\gamma_t \tag{20}$$

which is consistent with the dimensions of the involved flow variables.

Given a set of field-measured infiltration curves that characterize the infiltration process for a set of soil profiles, the parameters of the reference infiltration curve $I^*(t^*)$ and the respective sets of scaling factors γ_v, γ_I and γ_t can be determined using the following technique:

1. Each measured infiltration curve is approximated by *Philip*'s [1957] expression:

$$I(t) = St^{1/2} + At \tag{21}$$

2. The infiltration scaling factors γ_I and γ_t are derived by substituting (18) into (21), whereas γ_v is calculated directly from (20):

$$\gamma_I = \left(\frac{S}{S^*}\right)^2 \bigg/ \left(\frac{A}{A^*}\right) \qquad \gamma_t = \left(\frac{S}{S^*}\right)^2 \bigg/ \left(\frac{A}{A^*}\right)^2 \qquad \gamma_v = \frac{A}{A^*} \tag{22}$$

where S^* and A^* are the arithmetic means of the respective S and A values for each of the measured infiltration curves.

3. The scaling factors γ_K and γ_θ as determined from (13) have arithmetic means of 1. To satisfy relationship (19), the scaling factors γ_v and γ_I are divided by their respective mean values and the scaling factor γ_t is recalculated using (20). Consequently, scaling factors γ_v and γ_I have an arithmetic mean of 1 as well.

4. The normalization of scaling factors in step 3 requires new values for S^* and A^*. The expressions in (22) are used to find the parameters of the new reference curve:

$$S^*_{\text{new}} = \left(\overline{\gamma_I}\, \frac{A^*_{\text{new}}}{A^*}\right)^{1/2} S^* \qquad A^*_{\text{new}} = \overline{\gamma_v} A^* \tag{23}$$

where $\overline{\gamma_v}$ and $\overline{\gamma_I}$ are arithmetic means of the respective scaling factors before the normalization in step 3.

As an example we demonstrate the scaling of 36 field-measured infiltration functions. The double-ring ponded infiltration experiments were carried out on a fine sandy soil in the Hupselse Beek watershed, the Netherlands. The infiltration measurements were done on a regular square grid

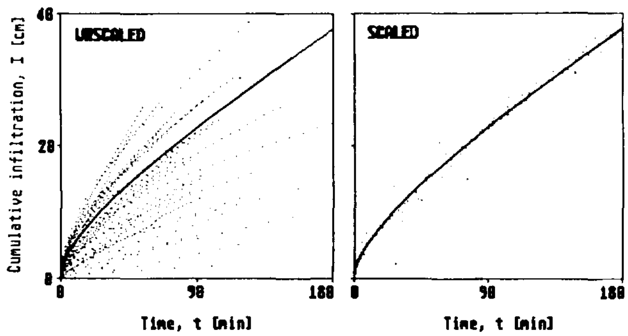

Fig. 4. Scaling of infiltration curves (Hupselse Beek data).

covering an area of 100 m × 100 m. Instantaneous infiltration rates were measured versus time until steady state conditions were approached. Infiltration data before and after linear scaling are shown in Figure 4. If Philip's formula is used, the calculated scaling factors provide exactly the same flexibility in describing the variability of a set of infiltration curves as do the original parameters S and A. Consequently, the variability of cumulative infiltration is always linear.

Relationships similar to those shown for the infiltration process can be found for other water flow problems. A scaling procedure is then applied to the respective dynamic characteristics of these processes, i.e., laboratory-determined outflow curves, field-measured drainage curves or pressure head and moisture content profiles during redistribution.

Scaling of Outflow Curves

As a last example we show how pressurized outflow curves may be used to estimate the variability of soil hydraulic properties. One hundred undisturbed soil samples from a 20-ha agricultural field near Los Banos, California [*Wallender and Hopmans*, 1990] were collected in 60-mm-deep and 80-mm-diameter soil cores. After saturation in Tempe pressure cells, these samples were subjected to 10 m of pressure. Cumulative outflow, Q, was measured as a function of time, t, during pressure desorption. These outflow data, together with independently determined saturated water content and the volumetric water content at a pressure head of -30 m (θ_{-30}), were input to a numerical model simulating the outflow process. The model optimizes the parameters of the soil hydraulic functions [*van Genuchten*, 1980] such that the difference between measured and simulated outflow data for each individual sample is minimized [*Kool et al.*, 1985]. The subsequent analysis was applied to the loamy soil samples only, comprising only 20 of the total number of samples.

The individual outflow data for these 20 samples are shown in Figure 5*a*. Each outflow curve results in a soil water retention and an unsaturated hydraulic conductivity curve with parameters obtained from the optimization model. Alternatively, we scaled the outflow curves prior to optimization, yielding γ_Q and γ_t values for each sample. The results are shown in Figure 5*b*. Scaling was done with the constraint that the average scaling factor of cumulative outflow (γ_Q) was equal to one. The reference outflow curve through the scaled outflow data and average values for the saturated water content and θ_{-30} were input to the optimization model. This yielded hydraulic functions of the reference soil. Subsequently, the variation of the hydraulic functions was determined from the scaling relations in (19), with I replaced by Q: $\gamma_Q = \gamma_\theta$ and $\gamma_t = \gamma_\theta/\gamma_K$. This could be done since the initial and boundary conditions fulfill the conditions in (12), and the variability is assumed to have a global component only ($\beta = 1$ and $\alpha = \gamma$). The hydraulic data computed from the scaled outflow curves are compared with the hydraulic data from optimization of each individual soil sample in Figure 6. Rather than comparing the hydraulic functions directly, we plotted individual volumetric water content and unsaturated hydraulic conductivity values for 10 soil water pressure head values, uniformly distributed between 0 and -10 m. Perfect scaling would result in data points falling along the 1:1 line. With the exception of two samples, application of the linear scaling concept described variability of soil hydraulic data well.

DISCUSSION AND APPLICATIONS

The assumptions on which the presented linear scaling theory is based lead to similar soil hydraulic properties and similar flow processes. The two similarities are connected by (6). In this respect, the theory is equivalent to the original *Miller and Miller* [1956] theory. However, the theoretical basis is more general and may lead to wider applications. The discussion hereafter demonstrates some of these applications.

The concept of similar soil properties can be used for soil classification. The linear transformation in (1) could provide a basis for classification of soils with each soil class being

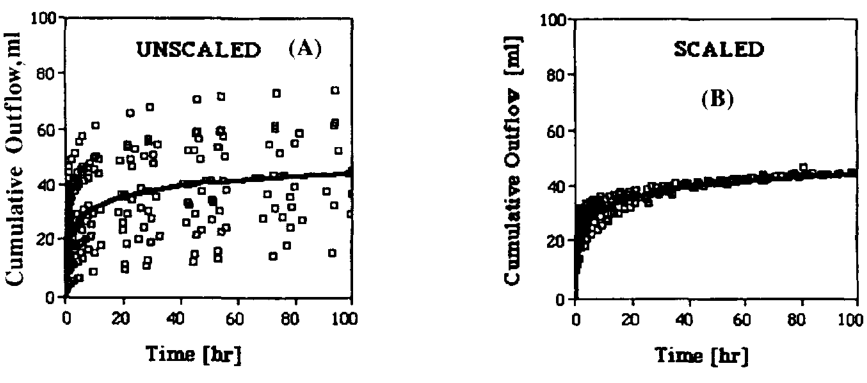

Fig. 5. Scaling of outflow curves. Symbols denote (*a*) unscaled and (*b*) scaled outflow data; the solid line is the reference curve.

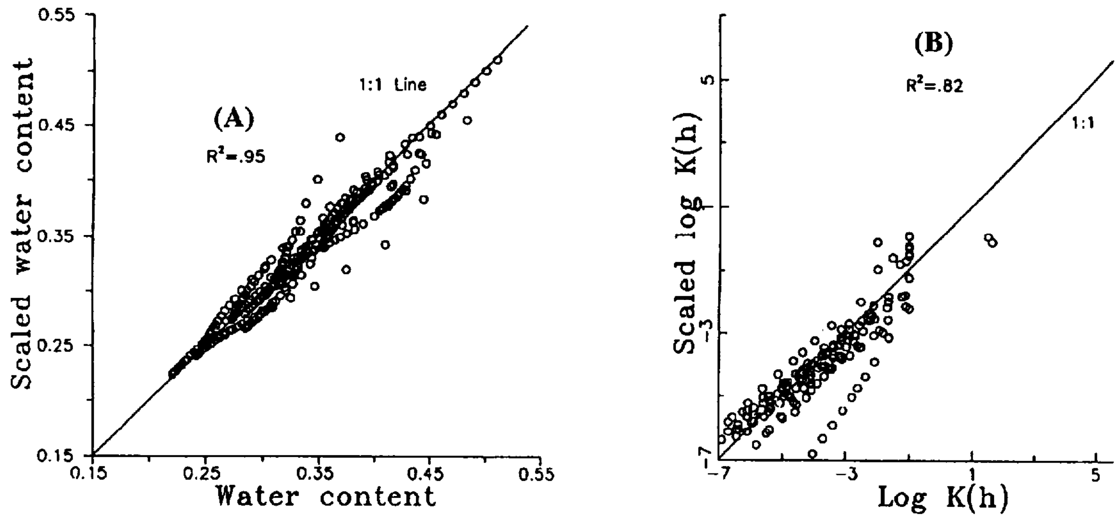

Fig. 6. Comparison of optimized and scaled (from outflow curves) (*a*) volumetric water content and (*b*) unsaturated hydraulic conductivity data.

identified by its reference hydraulic characteristics. For that purpose one may regard shape factors as the nonlinear parameters in the analytical expressions used for approximation of the soil water retention and hydraulic conductivity data. Different soil classes could then be defined by the shape factors of their reference hydraulic characteristics, and soil variability within each class could be expressed in terms of scaling factors. Shape parameters might correlate with soil texture, while scaling parameters are dominated by soil structure. The differentiation between scale and shape parameters corresponds to the decomposition of soil variability into its linear and nonlinear component. In *van Genuchten*'s [1980] hydraulic functions, α_{vg} and n relate to the scale and shape parameter, respectively.

The dynamic aspect of the linear variability concept can be used in computer simulations for the analysis of soil water flow. One can distinguish between two areas of application. The first group of applications includes problems for which the restrictive conditions (8) and (12) are satisfied. The second group applies to the remainder of numerical modeling applications. In approximating the soil variability by its linear component, soil profiles are replaced by fictitious linearly nonhomogeneous soils. Nevertheless, studying the behavior of a fictitious soil can yield important information about the original soil water system.

The first group of applications have one basic feature in common. There is no need to repeat the numerical simulation for a given problem if changes are made in saturated conductivity or saturated moisture content, no matter how complicated their space distribution is, so long as those changes are proportional throughout the soil profile. Relations (6) can be used instead to calculate a response to those changes. Consequently, a single simulation run of a numerical model gives probability distributions of output variables, provided that the probability distribution of input scaling factors is known.

Furthermore, relationships between scaling factors for soil hydraulic properties and those derived for dynamic characteristics can be very useful. Their use may lead to substantial savings of experimental and computational effort and simplify the analysis necessary to assess the consequences of the space and time variability of soil properties. The scaling of pressurized outflow curves is an example of such application. Direct measurement of dynamic characteristics is generally simpler and faster than "direct" determination of static soil hydraulic characteristics, which requires exhaustive and time-consuming measurements of long sequences of "equilibrium to equilibrium" processes.

The global component of linear variability as introduced in (2) could also be considered as virtual variability instead. The variability is then interpreted as an uncertainty in the determination of true "field scale" hydraulic properties by measurements based on "laboratory scale" soil samples. Scaling factors relate dynamic output variables obtained from numerical simulation to the corresponding field-measured variables and thus contain information on the relation between "laboratory scale" and "field scale" soil hydraulic properties.

The spectrum of applications is even more plentiful if conditions (8) and (12) are not required. The linear variability concept then serves as a tool to simplify the space and time variability encountered in the field. Stochastic-deterministic analysis of transient unsaturated and saturated flow in stratified soil profiles for complex time dependent boundary conditions is an example. In this group of applications, the relationships in (6) are no longer valid and simulation runs must be repeated when input parameters are changed. However, the linear variability concept will still provide us with a set of local and global scaling factors that simplify the parameterization of variable soil properties.

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